

Path integral measure factorization in path integrals for diffusion of Yang–Mills fields

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Abstract

Factorization of the (formal) path integral measure in a Wiener path integrals for Yang–Mills diffusion is studied.

Using the nonlinear filtering stochastic differential equation, we perform the transformation of the path integral defined on a total space of the Yang–Mills principal fiber bundle and come to the reduced path integral on a Coulomb gauge surface.

Integral relation between the path integral representing the “quantum” evolution given on the original manifold of Yang–Mills fields and the path integral on the reduced manifold defined by the Coulomb gauge is obtained.

1 Introduction

The gauge field theories belong to a class of infinite dimensional dynamical systems with a symmetry. One of the main problem in theoretical and mathematical physics is the quantization of such dynamical systems. A crucial question is of how the extra degrees of freedom should be treated in quantization of the gauge theories.

Every dynamical system with a symmetry give rises to a system with a lower degrees of freedom. A new system (a reduced dynamical system) may be completely described via invariant variables.

It can be assumed that similarly to the classical case a quantum behaviour of two systems (the original system and the reduced one) are also related to each other. This assumption is verified, for example, in path integral quantization of finite dimensional dynamical systems with a symmetry where

we have an integral relation between the corresponding path integrals for the original and the reduced dynamical systems.

In order to get the path integral for the reduced system in a gauge field theory, we make use of the Faddeev – Popov method [1]. In this method we consider the quantum evolution on a gauge surface. This evolution is equivalent to the quantum evolution of the dynamical system given on the gauge orbit space.

At present, the Faddeev – Popov method of the path integral quantization of the gauge theories is the most effective method for studying the reduced quantum evolution in the perturbation theory. In this quantization method, the special transformation of the path integrals cancels the redundant degrees of freedom that are related to the gauge symmetry.

Another method which can be used for description of the quantum evolution of the reduced system was proposed by Rossi and Testa in [2]. Almost the same approach to the quantization of the Yang–Mills field was given in the paper of Teitelboim [3]. In these methods, the quantum evolution on the orbit space of a group action was presented¹ by the integral over the gauge group with the original gauge field propagator as the integrand.

Exploring the relationship between the Faddeev–Popov quantization and that one given by Rossi and Testa, we carried out the model investigation of a path integral reduction problem in a finite dimensional case [5, 6]. We have considered the dynamical system describing the motion of a scalar particle on a smooth compact manifold with a given Lie group action. This dynamical system may be regarded as a finite dimensional model for a dynamical system with a gauge symmetry. A standard quantization of this model was presented in [7, 8].

In our papers, we have considered the diffusion of a scalar particle on a smooth compact manifold and have established that a path integral measure generated by the stochastic process does not invariant under reduction. The path integral transformation (from the original path integral to the path integral which describes the evolution of the reduced system) gave rise to additional terms (the transformation Jacobian) in the differential generator (the reduced "quantum Hamiltonian") of the semigroup related with the reduced stochastic process.

This result was obtained by factorization of a path integral measure. An initial path integral measure was decomposed into two measures. The first measure was generated by the stochastic process given on the orbit of the group action and the second measure was constructed by the stochastic process defined on the orbit space.

¹This evolution is locally equivalent to the evolution on a gauge surface.

The path integral measure transformation was performed by using the nonlinear filtering stochastic differential equation from the stochastic process theory. Due to the symmetry of our model, the nonlinear filtering equation was in fact the linear equation. This allowed us to present its solution via the multiplicative stochastic integral.

In this paper we extend our method of the local measure factorization [6] to the path integrals of the Euclidean Yang–Mills field theory. That is, considering the Schrödinger approach to the quantization of the field theories, we shall study the transformation of the path integral which represents the evolution of an initial function given on a space of Yang–Mills connections. As in [2], the noncanonical variable A_0 will be excluded from evolution by the gauge condition $A_0 = 0$. The residual gauge degrees of freedom are related to the time independent gauge transformations. We shall fix this rest symmetry by using the Coulomb gauge.

It is known that owing to an ambiguity of the Coulomb gauge, one can not uniquely determine the coordinates of a point on a total space of the principal fiber bundle. Nevertheless we assume in the paper that in restricted domain, to which our evolution belongs, it is possible to use this gauge for fixing the residual degrees of gauge freedom. Because of the local character of our evolution we shall not consider in the paper the effects coming from the nontrivial topology of the reduced manifold.

In a finite dimensional case, a free, proper and isometric action of a Lie group on a smooth compact manifold leads to a principal fiber bundle picture in which an original manifold P can be viewed as a total space of this principal fiber bundle. In a gauge theory, the original manifold is an infinite dimensional space of the Yang–Mills connections. In this case, in order to meet the requirements of the slice theorem by which one gets the principal fiber bundle structure on a manifold, one has to impose some additional restrictions both on gauge connections and on the gauge transformation group. These questions have been studied in detail in [9, 10, 11, 12]. The results of these investigations enable us by two possibilities.

According to the first possibility, one must use the pointed gauge group with the elements from the corresponding Sobolev class of functions. Elements of this group are restricted to satisfy the condition $g(x_0) = e$ at some fixed point x_0 of the manifold M . (Here e is an identity element of a gauge group.)

We shall follow in the paper the second possibility by which the points of a manifold \mathcal{P} are chosen to be the irreducible connections in the principal fiber bundle $P(M, G)$ in Sobolev class H_k , $k > 3$. Also, the quotient group of

the gauge transformation group by its center² is used as the transformation group acting on this manifold. We shall denote this quotient group by \mathcal{G} , with $g \in \mathcal{G}$ of H_{k+1} . Then, by the slice theorem it can be proved that the orbit space $\mathcal{M} = \mathcal{P}/\mathcal{G}$ is a Hilbert manifold.

Further restrictions that one has to impose onto the class of allowed gauge fields are related with path integrations. In a finite dimensional case, we have used the path integrals in which measures were generated by the solutions of the stochastic differential equations. The main contribution to the study of the stochastic processes for Yang–Mills fields has been done by Asorey and Mitter in [13]. In their papers, it was given a rigorous definition of the regularized stochastic process on a Hilbert manifold of the Yang–Mills orbits. They proved the Itô formula for the stochastic differentiation of the Yang–Mills stochastic processes.³ Besides, they have constructed the regularized semigroup and its infinitesimal generator for the quantum evolution on the orbit space.

These results were obtained by compactifying the three-dimensional space with a volume cut-off and introducing the ultraviolet regularization. An original plane metric given on the space of gauge connections was modified by inserting of the extra factors $(I + \Delta_A/\Lambda^2)^k$, where $\Delta_A = d_A d_A^* + d_A^* d_A$ is the Laplace operator and Λ is a cut-off parameter. Such a replacement of the original (weak) metric allowed them to obtain a family of gauge invariant (strong) Riemannian metrics given on the space of the gauge connections \mathcal{P} . Then, using the regularized metric, they have constructed the diffusion processes on the tangent space over the original manifold of the Yang–Mills connections and on the gauge orbit manifold \mathcal{M} .

In [13], the global stochastic processes on a manifold have been defined from local processes by a standard method based on the parallel displacement in the fiber bundle. Notice that there is another approach to the global definition of the stochastic process on a manifold. It was proposed by Belopolskaya and Daletskii in [16]. It is their method was used in our papers concerning the path integral reduction in the finite dimensional case. In this method the local stochastic processes, defined on charts of the tangent bundle, are mapped onto the manifold with the help of the exponential mapping. It follows that one can study the global stochastic evolution by means of the local evolutions given on charts of the manifold.

The aim of our present investigation is to establish the relation between the path integral which describes the quantum evolution given on the original manifold of the Yang–Mills connections and the path integral representing

²The center of the gauge group is a stabilizer at each irreducible connections.

³This formula for the processes in Banach and Hilbert spaces was also proved in [16].

the quantum evolution on the gauge orbit space. The investigation will be based on the approach to the path integral measure factorization developed earlier in [5, 6].

We shall consider the case when the Riemannian metric of the original manifold (the manifold of gauge connections) is plane. In this case in order to have a properly defined Gaussian measure and the Wiener process on a tangent space to the original manifold one has to describe an evolution with the help of the rigged space: $\mathcal{H}_+ \subset \mathcal{H} \subset \mathcal{H}_-$. A choice of the corresponding Sobolev classes of functions for these Hilbert spaces (the choice of a definite \mathcal{H}_-) is determined by the analytical restrictions that are necessary for the definition of diffusion processes (and the path integrals) related with the original evolution given in the principal fiber bundle. The diffusion processes in such spaces were studied in [16].

We note that the inductive limit of the chain of continuously and densely embedded Sobolev–Hilbert spaces of distributions (the Hilbert spaces \mathcal{H}_{-k}) formed by completion of the Schwartz space of functions \mathcal{S} with respect to the appropriate norms is the space $\mathcal{S}'(R^3) = \bigcup_{k=1}^{\infty} \mathcal{H}_{-k}(R^3)$ ⁴ of the Schrödinger representation of the quantum field theory. The quantum operators of this representation act in the space $L^2(\mathcal{S}', d\mu)$. In case of a free quantum field theory, the measure μ is a Gaussian measure.

In path integral approach to the quantization of the field theories, one of the main problems is a correct treatment of the interaction potential in the Feynman–Kac formula. Before using this formula in the path integral describing the evolution in the space of functions given on \mathcal{S}' , one has to regularize the potential term of the Hamiltonian. We remark that the consistent solution of the quantum evolution problem in this approach was only done for the limited number of the simple quantum field models.

On the other hand, the study of the quantum evolution with the regularized form of the Hamiltonian (as e.g. in [13]) results in the renormalization problem in the obtained final expression. We note that general solution of this problem in path integration is known at present only in the scope of the perturbation theory.

By this reason in the paper we shall study the particular case of the evolution which is given on the Hilbert manifold of the gauge fields equipped with the plane (unregularized) metric. It means, in fact, that we shall deal with the factorization of the formal path integral measure.

⁴ \mathcal{S}' is the Schwartz space of tempered distribution.

2 Backward Kolmogorov equation

The Feynman propagation kernel

$$G(A_b, t_b; A_a, t_a) = \langle A_b | e^{-iH(t_b - t_a)} | A_a \rangle$$

in $A_0 = 0$ gauge can be written symbolically as [2]:

$$G(A_b, t_b; A_a, t_a) = \int_{\substack{A(t_a, \mathbf{x}) = A_a(\mathbf{x}) \\ A(t_b, \mathbf{x}) = A_b(\mathbf{x})}} DA(t, \mathbf{x}) e^{iS(A_0=0)},$$

where the action S is

$$S = \int_{t_a}^{t_b} dt \int d^3x L,$$

with $L = \frac{1}{2g_0^2} \sum Tr(F_{\mu\nu} F^{\mu\nu})$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$.

We shall investigate not a true quantum evolution given by above Feynman path integral but the evolution generated by the corresponding diffusion process on the Riemannian manifold of Yang–Mills connections. The transition probability of this process can be defined by the solution of the corresponding backward Kolmogorov equation. The Green function of the backward Kolmogorov equation with the self-adjoint operator also satisfies the forward equation which, in turn, can be transformed into the Schrödinger equation for the Feynman propagation kernel by changing the parameter κ of the equation for i .

Notice that the transition from the Wiener path integrals representing the solution of the forward Kolmogorov equation to the Feynmann path integrals is an independent problem in path integration.

Our backward Kolmogorov equation has the following formal form:

$$\begin{cases} \left(\frac{\partial}{\partial t_a} + \frac{1}{2} \mu^2 \kappa \Delta_{\mathcal{P}}[A_a] + \frac{1}{\mu^2 \kappa} V[A_a] \right) \psi_{t_b}(A_a(\mathbf{x}), t_a) = 0 \\ \psi_{t_b}(A_b(\mathbf{x}), t_b) = \phi_0(A_b(\mathbf{x})), \end{cases} \quad (t_b > t_a). \quad (1)$$

In eq.(1), $A_a(\mathbf{x}) \equiv (A_a)_i^\alpha(\mathbf{x})$, $\mu^2 = \hbar g_0^2$, $\phi_0(A)$ is a given initial function of the gauge connection, κ is a real positive parameter. $\Delta_{\mathcal{P}}[A]$ is the Laplace operator given on the original plane Riemannian manifold \mathcal{P} of gauge connections:

$$\Delta_{\mathcal{P}}[A] = G^{(\alpha, i, x) (\beta, j, x')} \frac{\delta^2}{\delta A^{(\alpha, i, x)} \delta A^{(\beta, j, x')}} = \int d^3x k^{\alpha\beta} \delta_{ij} \frac{\delta^2}{\delta A_i^\alpha(\mathbf{x}) \delta A_j^\beta(\mathbf{x})},$$

where $G^{(\alpha,i,x)(\beta,j,x')} = \delta^{\alpha\beta} \delta^{ij} \delta^3(\mathbf{x} - \mathbf{x}')$. (Here and what is follows we assume summation over equal discrete indices and integration in case of equal continuous indices.) An invariant potential term $V[A]$ of the Yang–Mills Hamiltonian is

$$V[A] = \int d^3x \frac{1}{2} k_{\alpha\beta} F_{ij}^\alpha(\mathbf{x}) F^{\beta ij}(\mathbf{x}),$$

$k_{\alpha\beta} = c_{\mu\alpha}^\tau c_{\tau\beta}^\mu$ is the Cartan–Killing metric for a group G .

We see that owing to the singular behavior of $G^{(\alpha,i,x)(\beta,j,y)}$, there are two functional derivatives given at the same point in the Laplace operator of eq.(1). It may lead to divergency and the expression, as it stands, is not correctly defined.

There are various methods to get over this difficulty. It can be done, for example, by modifying the original plane metric $G_{(\alpha,i,x)(\beta,j,y)}$. In [13], an extra convergent factor was introduced into the original weak metric. Thereby the Hilbert manifold was equipped with a strong Riemannian metric and it became possible to determine the regularized Laplace operator which acts in the space of twice differentiable and bounded functions given on this manifold.

When one describes the evolution in rigged Hilbert spaces the domain of the properly defined Laplace operator is the space of functions given on the Hilbert space of distributions \mathcal{H}_- .⁵ The action of the Laplace operator in this space can be defined [15, 19] as trace (in \mathcal{H}) of the second derivative of these functions taken in \mathcal{H}_- and then restricted to \mathcal{H} . The kernel of the corresponding differential operator is usually written (in a symbolical form) by means of the variational derivatives. It is also possible to determine the Laplace operator by its action on cylindrical functions given on \mathcal{H}_- (see e.g. [20] or [21]).

We note that if one chooses a (weak) scalar products (as a basic scalar products) in the Hilbert spaces, that are the tangent spaces to the original manifold, then one needs to come to the rigged Hilbert spaces. The rigged Hilbert space given in the tangent space can be obtained by making use of the Hilbert space construction from [13]. The model space of the Hilbert manifold with the modified Riemannian metric obtained there can be taken as the Hilbert space \mathcal{H}_+ in the triple of the rigged spaces. This space and its adjoint space \mathcal{H}_- (with respect to \mathcal{H}) supplies the tangent spaces to the original manifold \mathcal{P} with the rigged structure $\mathcal{H}_+ \subset \mathcal{H} \subset \mathcal{H}_-$.

The obtained picture should be further generalized since in quantum field theory the gauge fields belong to the space of distributions.

⁵In case of the Schrödinger representation of the quantum field theory this operator acts in the space of functions on \mathcal{S}' .

Therefore, one has to deal with a manifold which is a Hilbert manifold modeled on the Sobolev–Hilbert space of distributions \mathcal{H}_- . An inductive limit of a chain of such manifolds could be regarded as “manifold of Schwartz distributions”. This object is not yet elaborated enough to be applied in our case, but we note that it is possible to determine (see ref.[22]) the “dual manifold” which is the projective limit of the corresponding Hilbert manifolds.

According to [16] the solution of the backward Kolmogorov equation for the diffusion on the Hilbert Riemannian manifold \mathcal{P} (provided that the coefficients of the equation are appropriately chosen) can be presented as a limit (under subdivision of the time interval) of the superposition of the local semigroups:

$$\psi_{t_b}(A_a(\mathbf{x}), t_a) = \hat{U}(t_b, t_a)\phi_0(p_a) = \lim_q \tilde{U}_\eta(t_a, t_1) \cdot \dots \cdot \tilde{U}_\eta(t_{n-1}, t_b)\phi_0(A_a(\mathbf{x})). \quad (2)$$

The local evolution semigroups \tilde{U}_η are determined by the equations:

$$\tilde{U}_\eta(s, t)\phi(A) = E_{s,A}\phi(\eta(t)) \quad s \leq t, \quad \eta(s, \mathbf{x}) = A(\mathbf{x}), \quad (3)$$

where the expectation value of the functions ϕ is taken over the stochastic process which is a local representative of the global stochastic process η_t obtained by means of the exponential mapping from the corresponding stochastic process defined in the tangent bundle $T\mathcal{P}$.

Thus the behaviour of the original global semigroup is determined by local evolution semigroups. And these local semigroups are defined by solutions of the local stochastic diffusion differential equations. Therefore, it is possible to derive the transformations of the global semigroup (2) by studying transformations of the local stochastic differential equations.

The global semigroup (2) can be written (symbolically) as

$$\begin{aligned} \psi_{t_b}(A_a(\mathbf{x}), t_a) &= E\left[\phi_0(\eta(t_b)) \exp\left\{\frac{1}{\mu^2\kappa} \int_{t_a}^{t_b} V[\eta(u)]du\right\}\right] \\ &= \int_{\Omega_-} d\mu^\eta(\omega) \phi_0(\eta(t_b)) \exp\{\dots\}, \end{aligned} \quad (4)$$

where $\eta_t(\mathbf{x}) = \eta(t, \mathbf{x})$ is a stochastic process given on a manifold \mathcal{P} (in our case – on a manifold of the original gauge connections). μ^η is a measure generated by this process on the path space $\Omega_- = \{\omega_t \equiv \omega(t, \mathbf{x}) : \omega(t_a, \mathbf{x}) = 0, \eta(t, \mathbf{x}) = A_a(\mathbf{x}) + \omega(t, \mathbf{x})\}$.

Such a representation for the solution of the Kolmogorov equation given by the path integral over the space of paths Ω_- is possible if on charts of the tangent bundle to the Hilbert manifold \mathcal{P} there exist the self-adjoint, positive operators of trace class. Regarding each of these operators (usually defined

by the quadratic form) as the covariance operator of a Gaussian measure one determines the Wiener processes and then the diffusion processes on charts of $T\mathcal{P}$. In fact, it is the case of [13], where the manifold \mathcal{P} was endowed with the regularized metric.

The stochastic differential equation for the local components $\eta_t^{(\alpha,i,x)}$ of the regularized stochastic process η_t is as follows: ⁶

$$d\eta^{(\alpha,i,x)}(t) = \mu\sqrt{\kappa} \mathfrak{X}_{\bar{M}}^{(\alpha,i,x)}(\eta(t)) dw^{\bar{M}}(t), \quad (5)$$

where $w^{\bar{M}}(t)$ is a Wiener process and the “matrix” $\mathfrak{X}_{\bar{M}}^{(\alpha,i,x)}$ is derived from the local equality: $\sum_{\bar{K}} \mathfrak{X}_{\bar{K}}^{(\alpha,i,x)} \mathfrak{X}_{\bar{K}}^{(\beta,j,y)} = G^{(\alpha,i,x)(\beta,j,y)}$. (The bar over indices indicates that these indices are to be taken as belonging to the Euclidean space.) The regularized form of the metric leads to the matrices $\mathfrak{X}_{\bar{M}}^{(\alpha,i,x)}$ that include the factors $(I + \Delta_A/\Lambda^2)^{-k/2}$ with $k > 3$ and even. The differential generator of the “true” stochastic process defined by the local stochastic differential equation, which has the diffusion term with the same coefficient as in equations (5) and also the corresponding drift term, is the Laplace–Beltrami operator of the manifold with the regularized metric.

In our formal approach we shall not deal with the regularization questions. It means that our original stochastic process is given on a Hilbert manifold \mathcal{P} with a plane Riemannian metric. That is, we endow the Hilbert manifold \mathcal{P} of the gauge fields in Sobolev class H_k ($k > 3$) with the Riemannian structure by choosing the (weak) L^2 scalar product in its model space – the Hilbert space \mathcal{H} . The stochastic process on \mathcal{P} is described by the local stochastic differential equation (5) in which we set $\mathfrak{X}_{\bar{M}}^{\bar{A}} = \delta_{\bar{M}}^{\bar{A}}$.

The process $w^{\bar{M}}(t)$ of equation (5) is a “cylindrical version” of the Wiener process that can be constructed in the space of paths with the values in some Hilbert space \mathcal{H}_- in which there exists a Gaussian measure.⁷ In the Hilbert space \mathcal{H} , in case of using the L^2 weak scalar product, we can only determine the cylindrical measure, since the identity operator being the correlation operator of the covariance given by this scalar product is not of the trace class.

The Wiener process w_t with the values in \mathcal{H}_- and which has the identity correlation operator in \mathcal{H} is called the canonical Wiener process. In appropriate Itô calculus its properties in the Hilbert space \mathcal{H} can be written as follows:

$$\mathbb{E} \left(dw_t(f) dw_t(g) \right) = dt (f, g)_{L^2}$$

⁶We present here only the diffusion part of the equation.

⁷The corresponding local stochastic differential equations for the processes with the values in \mathcal{H}_- must be considered in a weak sense.

$$\mathbb{E} \left(dw_t(f) \right) = 0,$$

or formally, as $\mathbb{E} (dw_t(\mathbf{x}) dw_t(\mathbf{y})) = dt \delta^3(\mathbf{x} - \mathbf{y})$ and $\mathbb{E} (dw_t(\mathbf{x})) = 0$.

Thus our stochastic differential equations (5) can be used only for definition of the cylindrical measures on the space of the paths that have their values in the Hilbert space \mathcal{H} .

Therefore we can consider the semigroup (2) only as a formal expression presented by the superposition of multiple integrals. These integrals are obtained from the cylindrical approximations of the local evolution semigroups and are equal to the integrals of the cylindrical functions. The integrals are taken over the corresponding cylindrical measures. The rigorous definition of the semigroup (2) acting in the space of functions given on the Hilbert space of distributions \mathcal{H}_- should be based on the regularization of the metric and the renormalization of the final expression.

The evolution semigroup (2) acts in the Banach space of bounded and continuous functions (functionals) given on the space of gauge fields. The space of functions can be supplied by the scalar product

$$(\psi, \psi) = \int_{\mathcal{P}} \bar{\psi}(A(\mathbf{x})) \psi(A(\mathbf{x})) \prod_{\mathbf{x}} dA_i^\alpha(\mathbf{x}), \quad (6)$$

which also has a formal sense, since the “measure” $\prod_{\mathbf{x}} dA_i^\alpha(\mathbf{x})$ is not defined as a Lebesgue measure.

It is worth to notice that in case of exploiting the rigged Hilbert spaces for description of the evolution, one can overcome the problem of the definition of the scalar product in the space of the “wave functions”. It can be done by using the Gaussian measure instead of the formal measure of the scalar product (6). But in this case, because of changing of the “natural” normalization of the wave functions, the self-adjoint Hamilton operator has to include an additional term [15].

3 Metric on \mathcal{P} in bundle coordinates

The gauge field A defined on M is the pull-back of a smooth connection one-form given on a trivial principle fiber bundle $P(M, G)$ ($P = M \times G$). Here M – is a compact three-dimensional space (S^3 or T^3) and G – is a compact simple Lie group. We shall consider not a space of all smooth connections but its subspace \mathcal{P} of the irreducible connections. For a connected manifold M , the holonomy group of each irreducible connection coincides with the group G .

The gauge transformation group acts on a space of the irreducible connections. But this action is not free. To obtain a free action one should factor out the group of gauge transformations by the transformations taking their values in the centre of the group. The resulting factor group, which we shall denote by \mathcal{G} , consists of such maps from M to G that belong to the Sobolev class H_{k+1} with $k > 3$.

From the slice theorem it follows [14] that \mathcal{P} may be viewed as a total space of a bundle of connections $\pi : \mathcal{P} \rightarrow \mathcal{M}$ in which the base $\mathcal{M} = \mathcal{P}/\mathcal{G}$ is the space of gauge orbits. The space \mathcal{P} can be locally presented as $\pi^{-1}(\mathcal{U}) \sim \mathcal{U} \times \mathcal{G}$, where \mathcal{U} is a neighbourhood of a point $\pi(p)$ on the base manifold \mathcal{M} . Note that existence of such a principle fiber bundle was proved in [14, 11, 12, 22].

We shall define the coordinates on this principal fiber bundle by extending an approach of [24] to the infinite dimensional case. This approach was used in [6] when we studied the quantum motion of a scalar particle on a manifold with a given action of the compact group Lie. Notice that introduction of the coordinates by method of [24] is similar to what has been done earlier in other papers concerning the gauge field quantization [25, 26, 27].

The fiber bundle coordinates of a point $p \in \mathcal{P}$ will be determined by gauge fixing method. We use the Coulomb gauge condition $\chi^\nu(A) \equiv \partial^k A_k^\nu(\mathbf{x}) = 0$, ($\nu = 1, \dots, N_G$) which is imposed on the gauge fields. It means that original coordinates " Q^A " (i.e., the gauge fields $A_i^\alpha(\mathbf{x})$ in our case) of a point $p \in \mathcal{P}$ can be expressed by means of the coordinates of the corresponding point given on a gauge surface Σ and a coordinates of a certain group element of a gauge group \mathcal{G} .

We shall use the following symbolical representation for the right action of the gauge group on the space \mathcal{P} of the irreducible connections:

$$\tilde{A}^{(\alpha, i, x)} = F^{(\alpha, i, x)}(A(\mathbf{x}), g(\mathbf{x})).$$

An explicit form of the transformation is given by

$$\tilde{A}_i^\alpha(\mathbf{x}) = \rho_\beta^\alpha(g^{-1}(x))A_i^\beta(\mathbf{x}) + u_\mu^\alpha(g(\mathbf{x}))\frac{\partial g^\mu(\mathbf{x})}{\partial \mathbf{x}^i},$$

where matrix $u_\mu^\alpha(g(\mathbf{x}))$ is analogous to the matrix $u_\mu^\alpha(g)$ which is one of the auxiliaries matrices of the compact Lie groups. The left and right auxiliaries matrices result from the differentiation of the group multiplication function (the multiplication table given in the space of group parameters) by the group elements.

Recall that matrix $v_\beta^\alpha(g)$ is equal to $\frac{\partial \tilde{\Phi}^\alpha(g, g_1)}{\partial g_1^\beta}|_{g_1=e}$ and matrix $u_\beta^\alpha(g)$ is inverse to $v_\beta^\alpha(g)$: $v_\beta^\alpha(g)u_\gamma^\alpha = \delta_\beta^\alpha$. Likewise, the matrix $\bar{v}_\beta^\alpha(g)$ is defined by $\bar{v}_\beta^\alpha(g) =$

$\frac{\partial \Phi^\alpha(g_1, g)}{\partial g_1^\beta} \big|_{g_1=e}$, and \bar{u}_β^α is its inverse matrix. The matrix $\rho_\beta^\alpha(g) = \bar{u}_\nu^\alpha(g) v_\beta^\nu(g)$ is a matrix of the adjoint representation of a group G . An inverse matrix $\rho_\beta^\alpha(g^{-1}) \equiv \bar{\rho}_\beta^\alpha(g)$ is defined by $\bar{\rho}_\beta^\alpha \rho_\mu^\beta = \delta_\mu^\alpha$.⁸

Let an arbitrary point p with the coordinates Q^A be given on a manifold \mathcal{P} . And let the action of a group \mathcal{G} be given on this manifold. With the help of the gauge surface Σ ($\chi^\alpha = 0$), which has a transverse intersection with the orbits of the group action, we can determine such a group element $g \in \mathcal{G}$ that takes the point p along the orbit to the corresponding point on Σ . The coordinates $g(Q)$ of this element g can be obtained by solving the following equation:

$$\chi^\alpha(F(Q, g^{-1}(Q))) = 0.$$

For the Coulomb gauge, this equation is as follows:

$$\partial^i(\mathbf{x}) \left[\rho_\beta^\alpha(g(\mathbf{x})) A_i^\beta(\mathbf{x}) - \rho_\nu^\alpha(g(\mathbf{x})) u_\sigma^\nu(g(\mathbf{x})) \frac{\partial g^\sigma(\mathbf{x})}{\partial \mathbf{x}^i} \right] = 0.$$

That is, in order to find the element $g(\mathbf{x})$, which takes A to the gauge surface $\chi^\alpha = 0$, we must solve this equation provided that the gauge field $A(\mathbf{x})$ is given.

The coordinates Q^* of the corresponding point on a submanifold Σ can be obtained from the solution of the equation $Q^{*A} = F^A(Q, g^{-1}(Q))$, where Q are the coordinates of the original point on the manifold \mathcal{P} . In our case, the coordinates Q^* are the dependent coordinates: the gauge connections $A_i^{\alpha}(\mathbf{x})$ are subjected to the gauge condition $\chi^\alpha(A^*) = 0$.

Therefore we have the bijective correspondence $A_i^\alpha(\mathbf{x}) \leftrightarrow (A_i^{*\alpha}(\mathbf{x}), g^\mu(\mathbf{x}))$ given by the gauge transformation

$$A_i^\alpha(\mathbf{x}) = \rho_\beta^\alpha(g^{-1}(\mathbf{x})) A_i^{*\beta}(\mathbf{x}) + u_\mu^\alpha(g(\mathbf{x})) \frac{\partial g^\mu(\mathbf{x})}{\partial \mathbf{x}^i}.$$

Of course, all this is valid if the equation for $g^\mu(\mathbf{x})$ has a unique solution for a given $A_i^{*\alpha}(\mathbf{x})$.

Notice that we are allowed to use the points of the surface Σ for coordinatization of the total space \mathcal{P} of the bundle of connections since there is a local isomorphism between the trivial principal bundle $\Sigma \times \mathcal{G} \rightarrow \Sigma$ and the principal bundle $P(\mathcal{M}, \mathcal{G})$ (for the last bundle we have locally $\pi^{-1}(\mathcal{U}) \sim \mathcal{U} \times \mathcal{G}$).

Using the Coulomb gauge, we shall consider the evolution in a sufficiently small neighbourhood of a point p given on the original manifold. It will be also assumed that the gauge surface has a transversal intersection with each

⁸Matrix $\rho_{(\beta, y)}^{(\alpha, x)} = \rho_\beta^\alpha(g(\mathbf{x})) \delta^3(\mathbf{x} - \mathbf{y})$ will be the matrix of the adjoint representation of the gauge transformation group \mathcal{G} .

gauge orbit. And hence we will assume the validity of the slice theorem in our case.

Changing $A_i^\alpha(\mathbf{x})$ for $(A_i^{*\alpha}(\mathbf{x}), g^\mu(\mathbf{x}))$, we chose new coordinates on the manifold \mathcal{P} (the total space of the principal fiber bundle $\pi : \mathcal{P} \rightarrow \mathcal{M}$). Now our aim is to get a new coordinate representation of the original Riemannian metric

$$ds^2 = G_{(\alpha,i,x)(\beta,j,y)} \delta A^{(\alpha,i,x)} \delta A^{(\beta,j,y)},$$

where

$$G_{(\alpha,i,x)(\beta,j,y)} = G\left(\frac{\delta}{\delta A_i^\alpha(\mathbf{x})}, \frac{\delta}{\delta A_j^\beta(\mathbf{y})}\right) = k_{\alpha\beta} \delta^{ij} \delta^3(\mathbf{x} - \mathbf{y}).$$

As in a finite dimensional case, it can be done if we make a replacement of variables provided that the fact of dependence of the gauge fields A^* will be taken into account. The "vector fields" transformation formula is a straightforward generalization of the corresponding formula from the finite dimensional case:

$$\begin{aligned} \frac{\delta}{\delta A^{(\alpha,i,x)}} &= \check{F}_{(\alpha,i,x)}^{(\mu,k,u)} N_{(\mu,k,u)}^{(\nu,p,v)}(A^*) \frac{\delta}{\delta A^{*(\nu,p,v)}} \\ &+ \check{F}_{(\alpha,i,x)}^{(\epsilon,m,z)} \chi_{(\epsilon,m,z)}^{(\mu,v)}(A^*) (\Phi^{-1})_{(\mu,v)}^{(\beta,u)}(A^*) \bar{v}_{(\beta,u)}^{(\sigma,p)}(g) \frac{\delta}{\delta g^{(\sigma,p)}}, \end{aligned} \quad (7)$$

where we have denoted by \check{F} the matrix which is inverse to the matrix $F_{(\alpha,i,x)}^{(\mu,k,u)}$ defined as follows:

$$F_{(\beta,j,y)}^{(\alpha,i,x)}[A, g] = \frac{\delta \tilde{A}^{(\alpha,i,x)}}{\delta A^{(\beta,j,y)}} = \rho_\beta^\alpha(g^{-1}(\mathbf{x})) \delta_j^i \delta^3(\mathbf{x} - \mathbf{y}).$$

It satisfies the relation:

$$F_{(\beta,j,y)}^{(\alpha,i,x)} \check{F}_{(\epsilon,k,z)}^{(\beta,j,y)} = \delta_\epsilon^\alpha \delta_k^i \delta^3(\mathbf{x} - \mathbf{z}).$$

Note that matrix $F_{(\alpha,i,x)}^{(\mu,k,u)}$ acts in the tangent space to the manifold \mathcal{P} .

In formula (7), by $N_{(\mu,k,u)}^{(\nu,p,v)}$, which is equal to

$$N_{(\beta,j,y)}^{(\alpha,i,x)} = \delta_{(\beta,j,y)}^{(\alpha,i,x)} - K_{(\mu,z)}^{(\alpha,i,x)} (\Phi^{-1})_{(\nu,u)}^{(\mu,z)} \chi_{(\beta,j,y)}^{(\nu,u)},$$

we have denoted the projection operator onto the subspace which is orthogonal to the Killing vector field $K_{(\alpha,y)}$. For our metric $G_{(\alpha,i,x)(\beta,j,y)}$, the Killing vector field $K_{(\alpha,y)}$ is

$$K_{(\alpha,y)} = K_{(\alpha,y)}^{(\mu,i,x)} \frac{\delta}{\delta A^{(\mu,i,x)}},$$

where

$$K_{(\alpha,y)}^{(\mu,i,x)}(A) = [(\delta_\alpha^\mu \partial^i(\mathbf{x}) + c_{\nu\alpha}^\mu A^{\tilde{\nu}i}(\mathbf{x})) \delta^3(\mathbf{x} - \mathbf{y})] \equiv [\mathcal{D}_\alpha^{\mu i}(A(\mathbf{x})) \delta^3(\mathbf{x} - \mathbf{y})]$$

(here $\partial_i(\mathbf{x})$ is a partial derivative with respect to x^i). These vector fields are obtained by taking the functional derivative of $F^{(\alpha,i,x)}$ with respect to $g(\mathbf{x})$ and then setting g to an identity. Our Killing vector fields

$$K_{(\alpha,y)} = (-\delta_\alpha^\mu \partial_i(\mathbf{y}) + c_{\nu\alpha}^\mu A_i^{\tilde{\nu}}(\mathbf{y})) \frac{\delta}{\delta A_i^\mu(\mathbf{y})} \equiv \tilde{\mathcal{D}}_{\alpha i}^\mu(A(\mathbf{y})) \frac{\delta}{\delta A_i^\mu(\mathbf{y})}$$

(there is no an integration with respect to \mathbf{y}) act in the space of functions that depend on the gauge connections.

The Faddeev–Popov matrix Φ is defined as follows:

$$\Phi_{(\mu,z)}^{(\nu,y)}[A] = K_{(\mu,z)}^{(\alpha,i,x)} \chi_{(\alpha,i,x)}^{(\nu,y)}.$$

For the Coulomb gauge, we have

$$\chi_{(\alpha,i,x)}^{(\nu,y)} = \delta_\alpha^\nu [\partial_i(\mathbf{y}) \delta^3(\mathbf{y} - \mathbf{x})].$$

Therefore, the matrix Φ (restricted to the gauge surface) is equal to

$$\Phi_{(\mu,z)}^{(\nu,y)}[A^*] = [(\delta_\mu^\nu \partial^2(\mathbf{y}) + c_{\sigma\mu}^\nu A_i^{*\sigma}(\mathbf{y}) \partial^i(\mathbf{y})) \delta^3(\mathbf{y} - \mathbf{z})]$$

or

$$\Phi_{(\mu,z)}^{(\nu,y)}[A^*] = [(\mathcal{D}[A^*] \cdot \partial)_\mu^\nu(\mathbf{y}) \delta^3(\mathbf{y} - \mathbf{z})].$$

An inverse matrix Φ^{-1} can be determined by the equation

$$\Phi_{(\mu,z)}^{(\nu,y)} (\Phi^{-1})_{(\sigma,u)}^{(\mu,z)}(\mathbf{y}, \mathbf{u}) = \delta_\sigma^\nu \delta^3(\mathbf{y} - \mathbf{u})$$

That is, it is the Green function for the Faddeev–Popov operator:

$$[\partial_i(\mathbf{y}) \mathcal{D}_\mu^{\nu i}[A(\mathbf{y})]] (\Phi^{-1})_{(\sigma,u)}^{(\mu,y)}(\mathbf{y}, \mathbf{u}) = \delta_\sigma^\nu \delta^3(\mathbf{y} - \mathbf{u}).$$

(The boundary conditions of this operator depend on a concrete choice of a base manifold M .) By a second group of variables, the Green function Φ^{-1} satisfies the following equation:

$$[-\tilde{\mathcal{D}}_\lambda^{\sigma i}[A(\mathbf{z})] \partial_i(\mathbf{z})] (\Phi^{-1})_{(\sigma,z)}^{(\mu,y)}(\mathbf{y}, \mathbf{z}) = \delta_\lambda^\mu \delta^3(\mathbf{y} - \mathbf{z}).$$

Notice that in the formula (7), the matrix Φ^{-1} , as well as the other terms of the projector N , is given on the gauge surface Σ .

In definition of the transformed metric, besides the operator N , we shall also use another projection operator $(P_\perp)^{(\alpha,k,x)}_{(\beta,m,y)}$. This operator performs the projection onto the tangent plane to the submanifold Σ and can be written symbolically as

$$(P_\perp)^{(\alpha,k,x)}_{(\beta,m,y)} = \delta^\alpha_\beta \left[\delta^k_m + \partial_m \frac{1}{(-\partial^2)} \partial^k \right] \delta^3(\mathbf{y} - \mathbf{x}).$$

Its definition and properties (together with the properties of the projection operator N) are given in Appendix.

In new coordinates, the metric $\tilde{G}_{\mathcal{AB}}(A^*, g)$ of the manifold \mathcal{P} is presented by the matrix:

$$\tilde{G}_{\mathcal{AB}}(A^*, g) = \begin{pmatrix} \tilde{G}_{(\alpha,i,x)(\beta,j,y)} & \tilde{G}_{(\alpha,i,x)(\gamma,y)} \\ \tilde{G}_{(\gamma,y)(\alpha,i,x)} & \tilde{G}_{(\alpha,x)(\beta,y)} \end{pmatrix}. \quad (8)$$

It has the following elements with respect to the basis $(\frac{\delta}{\delta A^*}, \frac{\delta}{\delta g})$:

$$\begin{aligned} \tilde{G}_{(\alpha,i,x)(\beta,j,y)} &= G_{(\tilde{\alpha},m,\tilde{x})(\tilde{\beta},n,\tilde{y})} (P_\perp)^{(\tilde{\alpha},m,\tilde{x})}_{(\alpha,i,x)} (P_\perp)^{(\tilde{\beta},n,\tilde{y})}_{(\beta,j,y)} \\ &= k_{\tilde{\alpha}\beta} \delta_{in} (P_\perp)^{(\tilde{\alpha},n,x)}_{(\alpha,j,y)}. \end{aligned}$$

The off-diagonal elements of this metric are

$$\begin{aligned} \tilde{G}_{(\alpha,i,x)(\gamma,y)} &= G_{(\tilde{\alpha},m,\tilde{x})(\tilde{\beta},n,\tilde{y})} (P_\perp)^{(\tilde{\beta},n,\tilde{y})}_{(\alpha,i,x)} K^{(\tilde{\alpha},m,\tilde{x})}_{(\mu,u)} \bar{u}^{(\mu,u)}_{(\gamma,y)} \\ &= k_{\tilde{\alpha}\beta} \delta_{mn} c_{\sigma\mu}^{\tilde{\alpha}} A^{*\sigma m}(v) (P_\perp)^{(\tilde{\beta},n,y)}_{(\alpha,i,x)} \bar{u}^\mu_\gamma(g(\mathbf{y})). \end{aligned}$$

The element of $\tilde{G}_{(\alpha,x)(\beta,y)}$ of the transformed metric is equal to

$$\begin{aligned} \tilde{G}_{(\alpha,x)(\beta,y)} &= \gamma_{(\mu,\tilde{x})(\nu,\tilde{y})} \bar{u}^{(\mu,\tilde{x})}_{(\alpha,x)} \bar{u}^{(\nu,\tilde{y})}_{(\beta,y)} \\ &= k_{\epsilon\sigma} \delta^{kl} \left[\tilde{\mathcal{D}}_{\mu k}^\epsilon [A^*(\mathbf{x})] \mathcal{D}_{\nu l}^\sigma [A^*(\mathbf{y})] \delta^3(\mathbf{x} - \mathbf{y}) \right] \bar{u}^\mu_\alpha(g(\mathbf{x})) \bar{u}^\nu_\beta(g(\mathbf{y})). \end{aligned}$$

By analogy with the finite dimensional case, the orbit metric $\gamma_{(\mu,x)(\nu,y)}$ can be defined as

$$\gamma_{(\mu,x)(\nu,y)} = K^{(\alpha,i,z)}_{(\mu,x)} G_{(\alpha,i,z)(\beta,j,u)} K^{(\beta,j,u)}_{(\nu,y)}.$$

Hence,

$$\gamma_{(\mu,x)(\nu,y)} = \int d^3u d^3v k_{\varphi\alpha} \delta^{kl} \delta^3(\mathbf{u} - \mathbf{v}) [\mathcal{D}_{\mu k}^\varphi(\mathbf{u}) \delta^3(\mathbf{u} - \mathbf{x})] [\mathcal{D}_{\nu l}^\alpha(\mathbf{v}) \delta^3(\mathbf{v} - \mathbf{y})].$$

Metric γ may be explicitly written in the following form:

$$\gamma_{(\mu,x)(\nu,y)} = k_{\varphi\alpha} \delta^{kl} [(-\delta_\mu^\varphi \partial_k(\mathbf{x}) + c_{\sigma\mu}^\varphi A_k^\sigma(\mathbf{x}))(\delta_\nu^\alpha \partial_l(\mathbf{y}) + c_{\kappa\nu}^\alpha A_l^\kappa(\mathbf{y})) \delta^3(\mathbf{x} - \mathbf{y})].$$

Matrix $\tilde{G}^{AB}(A^*, g)$,

$$\tilde{G}^{AB}(A^*, g) = \begin{pmatrix} \tilde{G}^{(\alpha, i, x) (\beta, j, y)} & \tilde{G}^{(\alpha, i, x) (\sigma, y)} \\ \tilde{G}^{(\sigma, y) (\alpha, i, x)} & \tilde{G}^{(\sigma, x) (\epsilon, y)} \end{pmatrix}, \quad (9)$$

is inverse to the matrix \tilde{G}_{AB} . The matrix elements of \tilde{G}^{AB} are given by

$$\begin{aligned} \tilde{G}^{(\alpha, i, x) (\beta, j, y)} &= G^{(\tilde{\alpha}, m, \tilde{x}) (\tilde{\beta}, n, \tilde{y})} N_{(\tilde{\alpha}, m, \tilde{x})}^{(\alpha, i, x)} N_{(\tilde{\beta}, n, \tilde{y})}^{(\beta, j, y)}; \\ \tilde{G}^{(\alpha, i, x) (\sigma, y)} &= G^{(\tilde{\alpha}, n, \tilde{x}) (\tilde{\beta}, m, \tilde{y})} N_{(\tilde{\alpha}, n, \tilde{x})}^{(\alpha, i, x)} \Lambda_{(\tilde{\beta}, m, \tilde{y})}^{(\nu, y)} \bar{v}_\nu^\sigma(g(\mathbf{y})). \end{aligned}$$

(In the previous formula there is no an integration with respect to \mathbf{y}).

A new term Λ is defined as follows:

$$\Lambda_{(\tilde{\beta}, m, \tilde{y})}^{(\nu, u)} = \chi_{(\tilde{\beta}, m, \tilde{y})}^{(\mu, \tilde{z})} (\Phi^{-1})_{(\mu, \tilde{z})}^{(\nu, u)}.$$

Its explicit representation is

$$\Lambda_{(\tilde{\beta}, m, \tilde{y})}^{(\nu, u)} = (-1) \left[\partial_m(\tilde{\mathbf{y}}) (\Phi^{-1})_{(\tilde{\beta}, \tilde{y})}^{(\nu, u)}(\mathbf{u}, \tilde{\mathbf{y}}) \right].$$

The matrix element $\tilde{G}^{(\sigma, x) (\epsilon, y)}$ of the matrix \tilde{G}^{AB} may be written as

$$\tilde{G}^{(\sigma, x) (\epsilon, y)} = G^{(\tilde{\alpha}, n, \tilde{x}) (\tilde{\beta}, m, \tilde{y})} \Lambda_{(\tilde{\alpha}, n, \tilde{x})}^{(\nu, x)} \Lambda_{(\tilde{\beta}, m, \tilde{y})}^{(\mu, y)} \bar{v}_\nu^\sigma(g(\mathbf{x})) \bar{v}_\mu^\epsilon(g(\mathbf{y}))$$

(there is no here an integration with respect to x and y).

Notice that matrix elements of \tilde{G}_{AB} and \tilde{G}^{AB} are restricted to the gauge surface, that is, they depend on A^* .

The matrices (8) and (9) are pseudo inverse to each other:

$$\tilde{G}^{AB} \tilde{G}_{BC} = \begin{pmatrix} (P_\perp)_{(\nu, m, v)}^{(\mu, k, u)} & 0 \\ 0 & \delta_{(\beta, y)}^{(\alpha, x)} \end{pmatrix}.$$

The determinant of the matrix (8) is equal to

$$\begin{aligned} (\det \tilde{G}_{AB}) &= \det G_{AB}(Q^*) \det \gamma_{\alpha\beta}(Q^*) (\det \chi \chi^\top)^{-1}(Q^*) (\det \bar{u}_\nu^\mu(a))^2 \\ &\times (\det \Phi_\beta^\alpha(Q^*))^2 \det' (P_\perp)_B^C(Q^*). \end{aligned}$$

An explicit form of this determinant is given by the following formula:

$$\det \tilde{G}_{AB} = \det k_{\alpha\beta} \det^{-1}(-\partial^2) \det^2(\bar{u}_\nu^\mu) \det^2(\partial \cdot \mathcal{D}_\beta^\alpha[A^*]) \det' P_\perp,$$

where

$$\det' (P_\perp)_{(\beta, m, y)}^{(\alpha, k, x)} = \det' \left(\delta_m^k + \partial_m \frac{1}{(-\partial^2)} \partial^k \right).$$

4 Transformation of the stochastic process and the semigroup

In the previous section, we have introduced new local coordinates on the manifold \mathcal{P} of the gauge connections. These coordinates have been obtained by means of transformation of the original variables.

A similar local transformation can be done for the stochastic process $\eta(t)$ and we come to another stochastic process $\zeta(t)$. The local components $(A_t^*(\mathbf{x}), g_t(\mathbf{x}))$ of the process $\zeta(t)$ can be expressed through the components of the process $\eta(t)$ by the gauge transformation formulae:

$$\eta^{(\alpha, i, x)}(t) = F^{(\alpha, i, x)}(A^*(t), g(t)).$$

The local processes are given on charts of the manifold \mathcal{P} . These processes correspond stochastically to each other in general domains of the charts intersections. By applying the method of [16], we can then construct the global stochastic process in that part of the manifold \mathcal{P} , where we are allowed to consider the Coulomb gauge as a true gauge.

To transform the global semigroup (2), we, first of all, make the transformations of the local semigroups (3). Our transformations of the local stochastic processes are the phase space transformation of the stochastic processes. It is well known that these transformations conserve the probabilities. Therefore, the transformations will leave invariant the path integral measures in our local semigroups.

Measures of our path integrals are generated by the stochastic processes defined by the solutions of the stochastic differential equations. After performing the transition to the process $\zeta(t)$, we come to a new semigroup represented in the form of mathematical expectation of initial function taken with respect to the measure generated by the transformed process.

We are interested in the stochastic differential equations for the local components $(A_t^*(\mathbf{x}), g_t(\mathbf{x}))$ of the process $\zeta(t)$. Let us recall how the similar stochastic differential equations have been obtained [6] in a finite dimensional case. In that case we considered the transformation of the stochastic process $\eta(t)$ given on a compact Riemannian manifold \mathcal{P} .

Let $Q^{*A}(t)$ be the components of the process $\zeta^A(t) = (Q^{*A}(t), g^\alpha(t))$. Since there is a bijective correspondence between Q^A and (Q^{*A}, g^α) , we can express the coordinates Q^{*A} in terms of the original coordinates Q^A :

$$Q^{*A} = F^A(Q, g^{-1}(Q)).$$

The same relation can be written for the components of the corresponding stochastic processes. Performing the stochastic differentiation of the random

variable $Q^{*A}(t)$ with the Itô formula, we get the following equation:

$$dQ^{*A}(t) = \frac{\partial Q^{*A}}{\partial Q^E} d\eta^E(t) + \frac{1}{2} \frac{\partial^2 Q^{*A}}{\partial Q^E \partial Q^C} < d\eta^E(t) d\eta^C(t) > . \quad (10)$$

If we substitute (5) (which in a general case has the corresponding drift term) for $d\eta^A(t)$ in (10) and replace then the stochastic variable $Q^A(t)$ by $Q^{*A}(t)$ and $g^\alpha(t)$ according to the formula $\eta^A(t) = F^A(Q^{*B}(t), g^\alpha(t))$, we shall obtain the drift and diffusion terms of the stochastic differential equation of the local process $Q^{*A}(t)$. Such a transformation performed with account of the invariance of the original metric, leads us to the following stochastic differential equation for the local process Q_t^* :

$$dQ_t^{*A} = \frac{1}{2} \mu^2 \kappa \left[-G^{LM}(Q_t^*) \Gamma_{LM}^E(F(Q_t^*, e)) N_E^A(Q_t^*) + G^{LM}(Q_t^*) N_{LM}^A(Q_t^*) \right] dt + \mu \sqrt{\kappa} N_C^A(Q_t^*) \mathfrak{X}_M^C(Q_t^*) dw^{\bar{M}}(t) \quad (11)$$

where Γ - are the Christoffel symbols and by N_{LM}^A we have denoted the partial derivative of N_L^A with respect to Q^{*M} .

The stochastic differential equation for the components of the process $\zeta(t)$ that are given on a group manifold \mathcal{G} of the principal fiber bundle $P(\mathcal{M}, \mathcal{G})$ can be obtained by using the similar transformations.

The stochastic differential equation (11) has been rewritten [6] in order to be appropriate for performing the reduction in the path integral. The drift term of this equation has been separated into two parts. The first part is responsible for the stochastic movement on the gauge surface Σ . The second one, denoted by $j_{II}^A(Q^*)$, is the projection of the mean curvature of the group orbit onto the tangent plane to Σ . The obtained stochastic differential equation was

$$dQ^{*A}(t) = \mu^2 \kappa \left(-\frac{1}{2} G^{EM} N_E^C N_M^B {}^H \Gamma_{CB}^A + j_I^A + j_{II}^A \right) dt + \mu \sqrt{\kappa} N_C^A \tilde{\mathfrak{X}}_M^C dw^{\bar{M}}, \quad (12)$$

where j_{II}^A is ⁹

$$j_{II}^A(Q^*) = -\frac{1}{2} \mu^2 \kappa G^{EU} N_E^A N_U^D \left[\gamma^{\alpha\beta} G_{CD} (\tilde{\nabla}_{K_\alpha} K_\beta)^C \right] (Q^*).$$

The j_{II}^A may also be written as follows:

$$j_{II}^A(Q^*) = -\frac{1}{2} \mu^2 \kappa N_C^A \left[\gamma^{\alpha\beta} (\tilde{\nabla}_{K_\alpha} K_\beta)^C \right] (Q^*).$$

⁹In [6], this term was written with a wrong sign.

Note that the first part of the drift term of the equation (12) includes [6] the Christoffel coefficient ${}^H\Gamma$ of the “horizontal” connection and the mean curvature j_I of the orbit space \mathcal{M} .

Let us consider the particular case when the original metric is plane, that is, $G_{AB} = \delta_{AB}$. This case is important for us because in the diffusion problem of the Yang–Mills fields we also have a plane metric given on the manifold of the gauge connections. For the plane metric the equation (11) can be written as follows:

$$dQ_t^{*A} = \frac{1}{2}\mu^2\kappa \left[-N_E^A G^{MC} K_{\beta C}^E \Lambda_M^\beta \right] dt + \mu\sqrt{\kappa}N_C^A \mathfrak{X}_M^C dw^{\bar{M}}(t). \quad (13)$$

(The terms on the right-hand side of this equation depend on Q_t^* .)

As in general case, we rewrite the coefficient of the drift term in the stochastic differential equation (13). Now we present the mean curvature j_{II} of this term as

$$j_{II}^A = \frac{1}{2}(\mu^2\kappa) N_C^A G^{BC} K_{\sigma B}^P \mathcal{A}_P^\sigma,$$

where we have denoted by \mathcal{A}_P^σ a new quantity

$$\mathcal{A}_P^\sigma(Q^*) = \gamma^{\sigma\mu}(Q^*) K_\mu^R(Q^*) G_{RP}(Q^*).$$

It is related to the natural connection one-form $\mathcal{A}_i^\sigma(x)$ that exists in the considered principal fiber bundle $\pi : \mathcal{P} \rightarrow \mathcal{P}/\mathcal{G}$, namely,

$$\mathcal{A}_i^\sigma(x) = \mathcal{A}_P^\sigma(Q^*(x)) Q^{*P}_i(x).$$

Here, x are independent coordinates on the orbit space $\mathcal{M} = \mathcal{P}/\mathcal{G}$, and $Q^{*P}_i(x)$ are the partial derivatives (with respect to x^i) of the functions $Q^*(x)$ that “resolve” the gauge condition $\chi^\alpha(Q^*(x)) = 0$.

The connection $\mathcal{A}_i^\sigma(x)$ was called the mechanical connection in the reduction problems of the classical mechanic [30]. Its analog in Yang–Mills quantization is called the Coulomb connection.

Using this connection, we rewrite equation (13) in the following form

$$dQ_t^{*A} = \frac{1}{2}\mu^2\kappa \left(N_E^A G^{BE} K_{\epsilon B}^L (\Lambda_L^\epsilon - \mathcal{A}_L^\epsilon) dt + j_{II}^A \right) dt + \mu\kappa^{1/2} N_C^A \mathfrak{X}_M^C dw^{\bar{M}}. \quad (14)$$

What we have done with the stochastic process and its stochastic differential equation in a finite dimensional case can also be carried out with the Yang–Mills stochastic process.

We may transform the local process $\eta_t^{(\alpha,i,x)}$ defined by the equation (5) to the local process $\zeta_t^{(\alpha,i,x)}$ by making use of the Itô formula¹⁰ written in

¹⁰ This formula for the stochastic differential was proved in [13] where the diffusion process in the space of the Yang–Mills connections was considered.

terms of the functional derivatives. As in the finite dimensional case, the process ζ_t has two components: $(A_t^{*(\beta,j,x)}, g_t^{(\mu,x)})$, that is, $A_t^{*(\beta,j,x)} \equiv A_t^{*\beta j}(\mathbf{x})$ and $g_t^{(\mu,x)} \equiv g_t^\mu(\mathbf{x})$.¹¹

Performing the necessary transformation, we come to the stochastic differential equation which looks like the analogous equation obtained in the finite dimensional case. Therefore, we shall use the equation (14) assuming now that the values of this equation have the generalized indices, that is, the indices have the discrete and the continuous components.

Thus, the equation (14) will be regarded as the symbolical representation of the “true” stochastic differential equation, the explicit form of which can be easily obtained by integrating over the repeated continuous indices. In the sequel, we shall also employ an analogous symbolical representations for the other stochastic differential equations. The main terms of these equations will be also presented in the explicit form.

As for the representation of the terms of the equation (14) written for diffusion of the Yang–Mills fields, the connection \mathcal{A}_B^α is given by the following expression:

$$\mathcal{A}_{(\beta,j,y)}^{(\alpha,x)} = [\mathcal{D}_{\mu j}^\varphi(A^*(\mathbf{y}))\gamma^{(\alpha,x)(\mu,y)}] k_{\varphi\beta}.$$

In the stochastic differential equations adapted to the Yang–Mills diffusion, $K_{\alpha B}^C$ denotes the functional derivative of $K_{(\alpha,x)}^{(\epsilon,m,z)}$ with respect to $A^{(\beta,j,y)}$:

$$K_{(\alpha,x)(\beta,j,y)}^{(\epsilon,m,z)} \equiv \frac{\delta}{\delta A^{(\beta,j,y)}} K_{(\alpha,x)}^{(\epsilon,m,z)} = \delta_j^m c_{\beta\alpha}^\epsilon \delta^3(\mathbf{z} - \mathbf{x}) \delta^3(\mathbf{z} - \mathbf{y}).$$

Summing on the generalized index α in the expression $\mathcal{A}^\alpha K_{\alpha B}^C$ of the eq.(14), we get

$$k_{\varphi\epsilon} c_{\beta\alpha}^\epsilon [\mathcal{D}_{\mu j}^\varphi(A^*(\mathbf{z})) \gamma^{(\alpha,y)(\mu,z)}] \big|_{z=y}.$$

This expression includes an “inverse matrix” $\gamma^{(\alpha,y)(\mu,z)}$ for matrix $\gamma_{(\mu,x)(\nu,y)}$. It can be defined by the following equation:

$$\gamma_{(\mu,x)(\nu,y)} \gamma^{(\nu,y)(\sigma,z)} = \delta_{(\mu,x)}^{(\sigma,z)} \equiv \delta_\mu^\sigma \delta^3(\mathbf{z} - \mathbf{x}).$$

Performing the integration with respect to \mathbf{y} on the left-hand side of this equation, we get

$$k_{\varphi\alpha} \delta^{kl} \tilde{\mathcal{D}}_{\mu k}^\varphi(A^*(\mathbf{x})) \mathcal{D}_{\nu l}^\alpha(A^*(\mathbf{x})) \gamma^{(\nu,x)(\sigma,z)} = \delta_\mu^\sigma \delta^3(\mathbf{z} - \mathbf{x}).$$

Thus, $\gamma^{(\nu,x)(\sigma,z)}$ is the Green function of the operator $(\tilde{\mathcal{D}}\mathcal{D})_{\mu\nu}$. In some of our expressions it will be denoted by $\gamma^{(\nu,x)(\sigma,z)}[A^*]$ in order that to stress its implicit dependence on A^* .

¹¹The existence of the process g_t can be proved by using an approach which is similar to that one developed for the stochastic process given on the gauge group [18].

Finally, the j_{II}^A – term of the stochastic equation can be rewritten as follows:

$$j_{II}^{(\alpha,i,x)} = -1/2 (\mu^2 \kappa) \int dz N_{(\epsilon,m,z)}^{(\alpha,i,x)} c_{\sigma\nu}^\epsilon [\mathcal{D}_\mu^{\sigma m}(A^*(y)) \gamma^{(\mu,y)(\nu,z)}] \Big|_{y=z}.$$

Note also that if we make use of an explicit expression of the projector N and perform the corresponding integrations (“the summation” over repeated generalized indices in eq.(14)) we shall come to a rather long expression. By this reason we shall keep this projector in its symbolical form during the course of our transformations.

The stochastic differential equation for the group-valued component g_t of the process $\zeta_t^{(\alpha,i,x)}$ can be obtained in the same way as it was done for the component A_t^* . The equation for the $g_t^{(\mu,x)} \equiv g_t^\mu(\mathbf{x})$ coincides by its form with the corresponding stochastic differential equation for the group component of the finite dimensional case:

$$\begin{aligned} dg_t^\alpha = & -\frac{1}{2}\mu^2\kappa \left[G^{RP} \Lambda_R^\sigma \Lambda_B^\beta K_{\sigma P}^B \bar{v}_\beta^\alpha - G^{CA} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_A^\beta) \bar{v}_\beta^\alpha \right. \\ & \left. - G^{MB} \Lambda_M^\epsilon \Lambda_B^\beta \bar{v}_\epsilon^\nu \frac{\delta}{\delta g^\nu} (\bar{v}_\beta^\alpha) \right] dt + \mu \sqrt{\kappa} \bar{v}_\beta^\alpha \Lambda_B^\beta \tilde{\mathfrak{X}}_M^B dw^{\bar{M}}. \end{aligned} \quad (15)$$

We shall also make use of this equation in the diffusion of the Yang–Mills fields taking into account the remarks that have already been done on the stochastic differential equation for the process A_t^* .

Notice that the equation (15) can also be written as follows:

$$\begin{aligned} dg_t^\alpha = & -\frac{1}{2}\mu^2\kappa \left[\left(2 G^{AC} \Lambda_B^\beta \Lambda_C^\epsilon K_{\epsilon C}^B - G^{AC} \Lambda_B^\beta \Lambda_A^\epsilon \Lambda_C^\varphi K_{\varphi M}^B K_\epsilon^M \right) \bar{v}_\beta^\alpha \right. \\ & \left. - G^{MB} \Lambda_M^\epsilon \Lambda_B^\beta \bar{v}_\epsilon^\nu \frac{\delta}{\delta g^\nu} (\bar{v}_\beta^\alpha) \right] dt + \mu \sqrt{\kappa} \bar{v}_\beta^\alpha \Lambda_B^\beta \tilde{\mathfrak{X}}_M^B dw^{\bar{M}}. \end{aligned} \quad (16)$$

The obtained stochastic differential equations, (14) and (16), determine the local process $\zeta_t^{(\alpha,i,x)}$ and, consequently, a new semigroup \tilde{U}_ζ . Since the process $\zeta_t^{(\alpha,i,x)}$ was obtained from the process $\eta_t^{(\alpha,i,x)}$ by the phase space transformation of the stochastic processes, we have

$$\tilde{U}_\eta(s, t) \phi_0(A) = E_{s,A}[\phi_0(\eta(t))] = E_{s,(A^*,g)}[\tilde{\phi}_0(\zeta(t))], \quad s \leq t, \quad \eta(s) = A(\mathbf{x}),$$

$\zeta(s) = (A^*(\mathbf{x}), g(\mathbf{x}))$ and $A(\mathbf{x})$ is related to the initial value of the process $\zeta(t)$ by the gauge transformation: $A(\mathbf{x}) = F(A^*(\mathbf{x}), g(\mathbf{x}))$.

Hence, we have the following equality for the local semigroups:

$$\tilde{U}_\eta(s, t) \phi_0(A) = \tilde{U}_\zeta(s, t) \tilde{\phi}_0(A^*, g).$$

The corresponding superposition of the local semigroups \tilde{U}_ζ leads, as in [16], to the global semigroup \tilde{U}_ζ of the process $\zeta(t)$:

$$\psi_{t_b}(A_a(\mathbf{x}), t_a) = \lim_q \tilde{U}_\zeta(t_a, t_1) \cdot \dots \cdot \tilde{U}_\zeta(t_{n-1}, t_b) \tilde{\phi}_0(A_a^*, g_a), \quad (17)$$

This global semigroup which is the transformation of the original semigroup (2) determines the transformed path integral.

The path integral defined by (17) will be symbolically written as

$$\psi_{t_b}(A_a(\mathbf{x}), t_a) = \mathbb{E} \left[\tilde{\phi}_0(\xi_\Sigma(t_b), g(t_b)) \exp \left\{ \frac{1}{\mu^2 \kappa} \int_{t_a}^{t_b} \tilde{V}(\xi_\Sigma(u)) du \right\} \right],$$

where $\xi_\Sigma(t_a) = A_a^*(\mathbf{x})$, $g(t_a) = g_a(\mathbf{x})$. Using the Itô formula (in its functional form), we get the following symbolical representation of the differential generator (the Hamilton operator) of the semigroup (17) related to the process $\zeta(t)$:

$$\begin{aligned} & \frac{1}{2} \mu^2 \kappa \left(G^{CD} N_C^P N_D^B \frac{\delta^2}{\delta A^{*P} \delta A^{*B}} + N_E^P G^{BE} K_{\epsilon B}^L (\Lambda_L^\epsilon - \mathcal{A}_L^\epsilon) \frac{\delta}{\delta A^{*P}} \right. \\ & + N_C^E G^{BC} K_{\sigma B}^P \mathcal{A}_P^\sigma \frac{\delta}{\delta A^{*E}} + G^{AB} \Lambda_A^\alpha \Lambda_B^\beta \bar{L}_\alpha \bar{L}_\beta - G^{RP} \Lambda_R^\sigma \Lambda_B^\alpha K_{\sigma P}^B \bar{L}_\alpha \\ & \left. + G^{CA} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_A^\alpha) \bar{L}_\alpha + 2 G^{BC} N_C^P \Lambda_B^\alpha \bar{L}_\alpha \frac{\delta}{\delta A^{*P}} \right) + \frac{1}{\mu^2 \kappa} V, \end{aligned}$$

where by \bar{L}_α we denote $\bar{L}_\alpha = \bar{v}_\alpha^\mu(g(\mathbf{x})) \frac{\delta}{\delta g^\mu(\mathbf{x})}$. Note also that all terms of the obtained differential generator (except for the \bar{L}_α) depend on $A^*(\mathbf{x})$.

An explicit form of the differential generator may be easily obtained from its symbolical expression as a result of the corresponding integration over the repeated continuous indices.

5 Path integral measure factorization

Since in our investigation we assume that the stochastic processes can be defined by the method of [16], we shall make use of the approach to the factorization of the path integral measure that has been developed in [6]. This approach is based on the application of the nonlinear filtering equation from the stochastic processes theory.

Taking into account the Markov property of the process $\zeta(t)$, we present each local semigroup \tilde{U}_ζ of the equation (17) as follows:

$$\tilde{U}_\zeta(s, t) \tilde{\phi}(A_0^*, g_0) = \mathbb{E} \left[\mathbb{E}[\tilde{\phi}(A^*(t), g(t)) \mid (\mathcal{F}_{A^*})_s^t] \right], \quad (18)$$

According to the optimal nonlinear filtering theory [28, 29], the conditional mathematical expectation of $\tilde{\phi}$ given the sub- σ -algebra \mathcal{F}_{A^*} ,

$$\hat{\tilde{\phi}}(A^*(t)) \equiv \mathbb{E} \left[\tilde{\phi}(A^*(t), g(t)) \mid (\mathcal{F}_{A^*})_s^t \right],$$

satisfies the certain stochastic differential equation which is called the nonlinear filtering equation. Solving this equation, we get an information on the stochastic process g_t provided that we observe the process A_t^* .

It is important that the coefficients of the stochastic differential equations of the processes g_t and A_t^* depend on the stochastic variables in the following way:

$$\begin{cases} dA_t^* = a_1(A_t^*, g_t, t) dt + X_1(A_t^*, t) dw_t \\ dg_t = a(A_t^*, g_t, t) dt + X(A_t^*, g_t, t) dw_t. \end{cases}$$

It can therefore be possible to derive the stochastic nonlinear filtering equation [28, 29] from the stochastic differential equations of the processes A_t^* and g_t :

$$\begin{aligned} d\hat{\tilde{\phi}}(A_t^*) &= \mathbb{E} \left[\tilde{\phi}_t + \tilde{\phi}_g a + \frac{1}{2} \tilde{\phi}_{gg} (XX^\top) \mid (\mathcal{F}_{A^*})_s^t \right] dt \\ &+ \mathbb{E} \left[\tilde{\phi} \{a_1 - \hat{a}_1\} + \tilde{\phi}_g (XX_1^\top) \mid (\mathcal{F}_{A^*})_s^t \right] (X_1 X_1^\top)^{-1} (dA_t^* - \hat{a}_1 dt), \end{aligned}$$

where $\hat{a}_1 = \mathbb{E}[a_1(A_t^*, g_t, t) \mid (\mathcal{F}_{A^*})_s^t]$ and $\tilde{\phi}_t$ is the partial derivative of the $\tilde{\phi}$ with respect to t , and $\tilde{\phi}_g$ – the corresponding derivative with respect to g .

Note that in our case the drift coefficient a_1 of the stochastic differential equation (14) does not depend on the group-valued process g_t .

From the symmetry of our problem it follows that the nonlinear filtering stochastic differential equation is a linear equation. Its symbolical form looks like an analogous equation of a finite dimensional case [6]:

$$\begin{aligned} d\hat{\tilde{\phi}}(A_t^*) &= -\frac{1}{2} \mu^2 \kappa \left(G^{RP} \Lambda_R^\sigma \Lambda_B^\beta K_{P\sigma}^B - G^{CA} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_A^\beta) \right) \\ &\times \mathbb{E} \left[\bar{L}_\beta \tilde{\phi} \mid (\mathcal{F}_{A^*})_s^t \right] dt + \frac{1}{2} \mu^2 \kappa G^{CB} \Lambda_C^\nu \Lambda_B^\kappa \mathbb{E} \left[\bar{L}_\nu \bar{L}_\kappa \tilde{\phi} \mid (\mathcal{F}_{A^*})_s^t \right] dt \\ &+ \mu \sqrt{\kappa} \Lambda_C^\beta \Pi_K^C \tilde{\mathcal{X}}_M^K \mathbb{E} \left[\bar{L}_\beta \tilde{\phi} \mid (\mathcal{F}_{A^*})_s^t \right] dw_t^{\bar{M}}, \end{aligned} \quad (19)$$

where by Π we denote the projection operator onto the "horizontal subspace". This operator is defined by the equation: $\Pi_B^A = \delta_B^A - K_\alpha^A \gamma^{\alpha\beta} K_\beta^C G_{CB}$ and has the following properties: $\Pi_B^A K_\alpha^B = 0$, $\Pi_E^C N_B^E = \Pi_B^C$, $\Pi_E^B N_B^T = N_E^T$.

Using the properties of the conditional mathematical expectations for the Markov processes, we could change the equation (19) for some solvable

equation. But, it could be done if we were able to factorise the variables A^* and g in our functional $\tilde{\phi}$. In a finite dimensional case, we have used the Peter–Weyl theorem. It allowed us to develop the function given on a compact group in series over the irreducible representations of this group.

In order to apply the similar approach to the problem considered in the present paper, we shall make use of the irreducible finite dimensional unitary representation constructed in [2]. This representation is given by the functional of the local group elements $g(\mathbf{x})$. For a compact group G , this functional depends on the values assumed by $g(\mathbf{x})$ in a finite number of points.

The matrices of the representation are written as follows:

$$Y^\lambda(g(x)) = \exp(i (J_\mu)^\lambda g^\mu(\mathbf{x})),$$

where $(J_\mu)^\lambda$ are the infinitesimal generators of the representation: $\bar{L}_\mu Y_{pq}^\lambda(g) = \sum_{q'} (J_\mu)_{pq'}^\lambda Y_{q'q}^\lambda(g)$.

We shall assume that it is possible to develop the functional $\tilde{\phi}$ in a series over these irreducible representation:

$$\tilde{\phi}(A^*, g) = \sum_{\lambda, p, q} r_{pq}^\lambda(A^*) Y_{pq}^\lambda(g).$$

Making use of such a representation for $\tilde{\phi}$ in the conditional mathematical expectation, we obtain

$$\mathbb{E}[\tilde{\phi}(A^*(t), g(t)) \mid (\mathcal{F}_{A^*})_s^t] = \sum_{\lambda, p, q} r_{pq}^\lambda(A^*(t)) \hat{Y}_{pq}^\lambda(A^*(t)), \quad (20)$$

where $\hat{Y}_{pq}^\lambda(A^*(t)) = \mathbb{E}[Y_{pq}^\lambda(g(t)) \mid (\mathcal{F}_{A^*})_s^t]$. Since r_{pq}^λ only depends on A_t^* , we have taken it out of the conditional expectation.

Notice that the conditional mathematical expectation $\hat{Y}_{pq}^\lambda(A^*(t))$ besides A_t^* depends also on initial values of the stochastic processes (i.e., it depends on $A_0^*(\mathbf{x}) = A_s^*(\mathbf{x})$ and $g_0^\alpha(\mathbf{x}) = g_s^\alpha(\mathbf{x})$).

Using the relation (20) in eq.(19), we get the following equation for \hat{Y}_{pq}^λ :

$$\begin{aligned} d\hat{Y}_{pq}^\lambda(A^*(t)) = & -\frac{1}{2}\mu^2\kappa \left\{ \left[G^{RP} \Lambda_R^\sigma \Lambda_B^\mu K_{P\sigma}^B - G^{CP} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_P^\beta) (J_\beta)_{pq'}^\lambda \hat{Y}_{q'q}^\lambda(A^*(t)) \right] \right. \\ & \left. - G^{CB} \Lambda_C^\alpha \Lambda_B^\nu (J_\alpha)_{pq'}^\lambda (J_\nu)_{q'q''}^\lambda \hat{Y}_{q''q}^\lambda(A^*(t)) \right\} dt \\ & + \mu \sqrt{\kappa} \Lambda_C^\nu \Pi_K^C (J_\nu)_{pq'}^\lambda \hat{Y}_{q'q}^\lambda(A^*(t)) \tilde{\mathcal{X}}_M^K(A^*(t)) dw^{\bar{M}}(t). \end{aligned} \quad (21)$$

This stochastic differential equation is a matrix linear equation. Its solution may be written [31] via the multiplicative stochastic integral:¹²

$$\hat{Y}_{pq}^\lambda(A^*(t)) = (\overline{\text{exp}})_{pn}^\lambda(A^*(t), t, s) \mathbb{E}[Y_{nq}^\lambda(g_s(\mathbf{x})) \mid (\mathcal{F}_{A^*})_s^t], \quad (22)$$

where

$$\begin{aligned} (\overline{\text{exp}})_{pn}^\lambda(A^*(t), t, s) = & \overline{\text{exp}} \int_s^t \left\{ \frac{1}{2} \mu^2 \kappa \left[\bar{\gamma}^{\sigma\nu}(A^*(u)) (J_\sigma)_{pr}^\lambda (J_\nu)_{rn}^\lambda \right. \right. \\ & - \left(G^{RP} \Lambda_R^\sigma \Lambda_B^\beta K_{P\sigma}^B - G^{CA} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_A^\beta) \right) (J_\beta)_{pn}^\lambda \Big] du \\ & \left. + \mu \sqrt{\kappa} \Lambda_C^\beta (J_\beta)_{pn}^\lambda \Pi_K^C \tilde{\mathcal{X}}_M^K dw^{\bar{M}}(u) \right\}. \end{aligned} \quad (23)$$

Notice that for the conditional expectation taken at the initial values we have the equalities:

$$\mathbb{E} \left[Y_{nq}^\lambda(g_s(\mathbf{x})) \mid (\mathcal{F}_{A^*})_s^t \right] = Y_{nq}^\lambda(g_s(\mathbf{x})) = Y_{nq}^\lambda(g_0(\mathbf{x})).$$

Thus, the local semigroup (18) may be written as follows:

$$\tilde{U}_\zeta(s, t) \tilde{\phi}(A_0^*(\mathbf{x}), g_0(\mathbf{x})) = \sum_{\lambda, p, q, q'} \mathbb{E} \left[r_{pq}^\lambda(A_t^*) (\overline{\text{exp}})_{pq'}^\lambda(A_t^*, t, s) \right] Y_{q'q}^\lambda(g_0(\mathbf{x})). \quad (24)$$

If we make the similar transformations in all local semigroups that determine the global semigroup (17), we come to the following representation:

$$\psi_{t_b}(p_a, t_a) = \sum_{\lambda, p, q, q'} \mathbb{E} \left[c_{pq}^\lambda(\xi_\Sigma(t_b)) (\overline{\text{exp}})_{pq'}^\lambda(\xi_\Sigma(t), t_b, t_a) \right] Y_{q'q}^\lambda(g_a) \quad (25)$$

($\xi_\Sigma(t_a) = \pi|_\Sigma \circ p_a = A_a^*(\mathbf{x})$), where $\xi_\Sigma(t)$ is a global stochastic process given on a submanifold Σ . The local components of the process $\xi_\Sigma(t)$ are the stochastic processes A_t^* .

Let us assume that there exists a fundamental solution $G_{\mathcal{P}}(p_b, t_b; p_a, t_a)$ of our equation (1), that is,

$$\psi_{t_b}(p_a, t_a) = \int G_{\mathcal{P}}(p_b, t_b; p_a, t_a) \phi_0(p_b) dv_{\mathcal{P}}(p_b), \quad (26)$$

where $dv_{\mathcal{P}}$ is a “volume measure” on \mathcal{P} . It is then possible to transform (25) into the relation between the kernels (the Green functions) of the corresponding semigroups, provided that we choose the delta-function as an initial function of our original semigroup.

¹²The multiplicative stochastic integral is defined as a limit of the sequence of time-ordered multipliers that have been obtained as a result of breaking the time interval $[s, t]$. The direction of the arrow means that the order is taken from s to t .

The Green function $G_{\mathcal{P}}$, related to the Feynmann propagator, may be expressed as a sum of the the matrix Green function G_{mn}^λ multiplied by the matrix of the irreducible representation Y_{mn}^λ . Such a representation give us the relation between two path integrals. One of the path integrals describes the stochastic evolution on the manifold \mathcal{P} of the gauge connections, another one – represents the evolution given on the gauge surface Σ defined by the dependent variables A^* .

The kernel of the evolution semigroup (the Green function G_{mn}^λ) given by the mathematical expectation standing under the sign of the sum in (25) can be symbolically written as follows:

$$\begin{aligned}
& G_{mn}^\lambda(\pi_\Sigma(p_b), t_b; \pi_\Sigma(p_a), t_a) = \\
& \tilde{\mathbb{E}}_{\substack{\xi_\Sigma(t_a)=\pi_\Sigma(p_a) \\ \xi_\Sigma(t_b)=\pi_\Sigma(p_b)}} \left[(\tilde{\text{exp}})_{mn}^\lambda(\xi_\Sigma(t), t_b, t_a) \exp \left\{ \frac{1}{\mu^2 \kappa} \int_{t_a}^{t_b} \tilde{V}(\xi_\Sigma(u)) du \right\} \right] \\
& = \int_{\substack{\xi_\Sigma(t_a)=\pi_\Sigma(p_a) \\ \xi_\Sigma(t_b)=\pi_\Sigma(p_b)}} d\mu^{\xi_\Sigma} \exp \left\{ \frac{1}{\mu^2 \kappa} \int_{t_a}^{t_b} \tilde{V}(\xi_\Sigma(u)) du \right\} \\
& \times \tilde{\text{exp}} \int_{t_a}^{t_b} \left\{ \frac{1}{2} \mu^2 \kappa \left[\gamma^{\sigma\nu}(\xi_\Sigma(u)) (J_\sigma)_{mr}^\lambda (J_\nu)_{rn}^\lambda \right. \right. \\
& + G^{RP} \Lambda_R^\sigma \Lambda_B^\beta K_{P\sigma}^B - G^{CP} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_P^\beta) \left. \left. (J_\beta)_{mn}^\lambda \right] du \right. \\
& \left. + \mu \sqrt{\kappa} \Lambda_C^\beta (J_\beta)_{mn}^\lambda \Pi_K^C \tilde{\mathcal{X}}_M^K dw^{\bar{M}}(u) \right\}, \tag{27}
\end{aligned}$$

($\pi_\Sigma(p_b) = A_b^*(\mathbf{x})$ and $\pi_\Sigma(p_a) = A_a^*(\mathbf{x})$). In eq.(27), the path integral measure μ^{ξ_Σ} is generated by the process $\xi_\Sigma(t)$. The local components of this process are the solutions of the stochastic differential equation (14).

The differential generator of the semigroup given by eq.(27) is

$$\begin{aligned}
& \frac{1}{2} \mu^2 \kappa \left\{ \left[G^{CD} N_C^A N_D^B \frac{\delta^2}{\delta A^{*A} \delta A^{*B}} \right. \right. \\
& + N_E^A G^{BE} K_{\epsilon B}^L \Lambda_L^\epsilon \frac{\delta}{\delta A^{*A}} \left. \left. (I^\lambda)_{pq} + 2 N_C^A G^{CP} \Lambda_P^\alpha (J_\alpha)_{pq}^\lambda \frac{\delta}{\delta A^{*A}} \right. \right. \\
& - \left(G^{RP} \Lambda_R^\sigma \Lambda_B^\alpha K_{P\sigma}^B - G^{CA} N_C^M \frac{\delta}{\delta A^{*M}} (\Lambda_A^\alpha) \right) (J_\alpha)_{pq}^\lambda \\
& \left. + G^{SB} \Lambda_B^\alpha \Lambda_S^\sigma (J_\alpha)_{pq'}^\lambda (J_\sigma)_{q'q}^\lambda \right\} + \frac{1}{\mu^2 \kappa} V. \tag{28}
\end{aligned}$$

((I^λ) $_{pq}$ – is an identity matrix.)

This operator acts in the space of sections $\Gamma(\Sigma, V^*)$ of the corresponding covector bundle which is associated to the principal fiber bundle $\pi : \Sigma \times \mathcal{G} \rightarrow$

Σ . As in a finite dimensional case [6], it can be shown that the scalar product (6) is transformed to the formal scalar product in the space $\Gamma(\Sigma, V^*)$ given by

$$(\psi_n, \psi_m) = \int \langle \psi_n, \psi_m \rangle_{V_\lambda^*} \det \Phi_\beta^\alpha \prod_{\alpha=1}^{N_G} \delta(\chi^\alpha(A^*)) \prod_{\mathbf{x}} dA^*(\mathbf{x}),$$

where $\chi^\alpha(A^*) = \partial^k A_k^{\alpha}(\mathbf{x})$ and $\det \Phi_\beta^\alpha = \det(\partial \cdot \mathcal{D}_\beta^\alpha[A^*])$ is the determinant of the Faddeev–Popov operator in the Coulomb gauge.

In order to inverse the “global” relation between the Green functions which can be derived from (25) it needs to assume the existence of the partition of unity for the corresponding local covering of the manifold \mathcal{P} . Provided that this assumption takes place, one may recover the necessary integral relation between the “global” Green functions from the integral relations obtained by inversion of the local Green functions given on charts of this partition. The latter may be done by using the orthogonal relations of the matrix elements of the irreducible representation from [2] and properties of the transition functions of the principal fiber bundle. As a result one gets the following integral relation:

$$G_{mn}^\lambda(\pi_\Sigma(p_b), t_b; \pi_\Sigma(p_a), t_a) = \int_{\mathcal{G}} G_{\mathcal{P}}(p_b g, t_b; p_a, t_a) Y_{nm}^\lambda(g(\mathbf{x})) d\mu(g(\mathbf{x})), \quad (29)$$

where $d\mu(g(\mathbf{x}))$ is the normalized invariant volume element given on a group \mathcal{G} and by “ $p_b g$ ” we have denoted the gauge transformed boundary “point” $A_b(\mathbf{x})$. The obtained integral relation is defined (formally) on that domain of the manifold \mathcal{P} where using of the Coulomb gauge does not violate the slice theorem. A similar integral relation between Green functions was obtained in [2, 3].

In order to get the path integral representation for the kernel of the evolution semigroup acting in the space of the scalar functions given on a submanifold Σ one must set $\lambda = 0$ in eq.(27). This converts the multiplicative stochastic integral into the identity matrix. And the differential generator of the obtained semigroup will be the diagonal part of the Hamiltonian (28).

The path integral measure of the $\lambda = 0$ case is generated by the stochastic process defined by the local stochastic differential equations (14). But the equations of such a form have an “extra” term j_{II} . The differential generator of the process determined by the same local stochastic differential equations, but without these extra terms, would be a Laplace–Beltrami operator for the submanifold Σ . The diffusion on Σ governed by the Laplace–Beltrami operator is directly related to the diffusion on the gauge orbit space.

Therefore we make use of the Girsanov transformation in order to get a necessary description of the evolution on a gauge surface Σ . The Girsanov

transformation changes the path integral measure $\mu^\xi \equiv \mu^1$ generated by the process ξ_t whose local stochastic differential equations are

$$dA_t^{*C} = \frac{1}{2} \mu^2 \kappa \left(N_E^C G^{BE} K_{\epsilon B}^L (\Lambda_L^\epsilon - \mathcal{A}_L^\epsilon) dt + j_{II}^A \right) dt + \mu \kappa^{1/2} N_D^C \mathcal{X}_M^D dw_t^{\bar{M}}$$

for the path integral measure $\mu^2 \equiv \mu^{\tilde{\xi}_\Sigma}$ related to the stochastic process $\tilde{\xi}_\Sigma(t)$ with the local equations

$$dA_t^{*C} = \frac{1}{2} \mu^2 \kappa N_E^C G^{BE} K_{\epsilon B}^L (\Lambda_L^\epsilon - \mathcal{A}_L^\epsilon) dt + \mu \kappa^{1/2} N_D^C \mathcal{X}_M^D dw_t^{\bar{M}}.$$

The Jacobian of the transformation is given by the following general formula:

$$\frac{d\mu^1}{d\mu^2} = \exp \left\{ -\frac{1}{2} \int_{t_a}^{t_b} [A^{-1}(b-a)]^2 dt + \int_{t_a}^{t_b} (A^{-1}(b-a), dw_t) \right\}.$$

As in [6], in our case we will have

$$\begin{aligned} \frac{d\mu^1}{d\mu^2}(\tilde{\xi}_\Sigma(t)) = \exp \left\{ \int_{t_a}^t \left[-\frac{1}{2} \mu^2 \kappa (P_\perp)_A^L G_{LK}^H (P_\perp)_E^K j_{II}^A j_{II}^E \right] dt + \right. \\ \left. + \mu \kappa^{1/2} G_{LK}^H (P_\perp)_A^L j_{II}^A \mathcal{X}_M^K dw_t^{\bar{M}} \right\}, \end{aligned}$$

since

$$(b-a)^A = -j_{II}^A = \frac{1}{2} \gamma^{\nu\sigma} [\nabla_{K_\nu} K_\sigma]^C N_C^A.$$

The term of the Jacobian with the stochastic integral may be transformed for the following expression:

$$-\frac{1}{2} \mu \kappa^{1/2} \int_{t_a}^{t_b} G_{PC}^H \gamma^{\nu\sigma} [\nabla_{K_\nu} K_\sigma]^C \mathcal{X}_M^P dw_t^{\bar{M}}.$$

The Jacobian of the Girsanov transformation can also be written as follows:

$$\begin{aligned} \frac{d\mu^1}{d\mu^2} = \exp \left\{ \int_{t_a}^{t_b} \left[-\frac{1}{8} \mu^2 \kappa G^{RB} N_B^L N_R^M (\mathcal{A}_P^\alpha K_{\alpha L}^P) (\mathcal{A}_D^\beta K_{\beta M}^D) \right] dt + \right. \\ \left. + \frac{1}{2} \mu \kappa^{1/2} N_K^L (\mathcal{A}_C^\nu K_{\nu L}^C) \mathcal{X}_M^K dw_t^{\bar{M}} \right\}. \end{aligned}$$

We see that the Jacobian can be presented in terms of the projection operators and the "Coulomb connection" \mathcal{A}_P^ν .

Thus, for the kernel of the evolution semigroup, we get

$$\begin{aligned}
G_\Sigma(A_b^*, t_b; A_a^*, t_a) &= \int_{\substack{\tilde{\xi}_\Sigma(t_a)=A_a^* \\ \tilde{\xi}_\Sigma(t_b)=A_b^*}} d\mu^{\tilde{\xi}_\Sigma} \exp \left\{ \frac{1}{\mu^2 \kappa} \int_{t_a}^{t_b} V(\tilde{\xi}_\Sigma(u)) du \right\} \\
&\exp \left\{ \int_{t_a}^{t_b} \left[-\frac{1}{8} \mu^2 \kappa G^{RB} N_B^L N_R^M (\mathcal{A}_P^\alpha K_{\alpha L}^P) (\mathcal{A}_D^\beta K_{\beta M}^D) \right] dt + \right. \\
&\left. + \frac{1}{2} \mu \kappa^{1/2} N_K^L (\mathcal{A}_C^\nu K_{\nu L}^C) \mathcal{X}_M^K dw_t^{\bar{M}} \right\}, \\
&(A^* = \pi_\Sigma(p)).
\end{aligned}$$

6 Conclusion

Using the path integral measure factorization method, we have considered the separation of the physical and unphysical degrees of freedom in a pure Yang–Mills theory.

Our path integrals have been formally defined by taking the limits in the expressions obtained by the cylindrical approximations of the local semigroups that have been used by Belopolskaya and Daletskii [16] in their definition of the path integrals given on a Hilbert manifold. This definition is based on local stochastic processes given on charts of a manifold. In our case we have used the (weak) Riemannian metric on the Hilbert manifold. Therefore we were able to deal only with the cylindrical versions of the true local stochastic processes which have their values in the Hilbert space of distribution.

The factorization of the path integral measure has been performed with the help of the nonlinear filtering stochastic differential equation from the stochastic process theory. As a result of our transformation we have obtained an integral relation (29) between the fundamental solutions (the Green functions) of the backward Kolmogorov equations that are given on the original manifold and on the manifold Σ defined by the Coulomb gauge $\partial^k A_k^*(\mathbf{x}) = 0$. Both the left-hand side and the integrand of the right-hand side of this integral relation have been presented in terms of the corresponding path integrals.

Considering the reduction onto the zero-momentum level (the $\lambda = 0$ case), we have obtained the path integral representation of the Green function G_Σ . The integrand of the path integral representing G_Σ besides the potential term consists of the transformation Jacobian which has the functional dependence on the Coulomb connection.

The Green function G_Σ is an Euclidean analog of the Feynman propagator which is used to describe the quantum evolution on the orbit space. More precisely, it gives us an implicate description (in terms of dependent

gauge fields) of the gauge-invariant modes that could be associated with the “mysterious” glueball particles and their excitations when $\lambda \neq 0$.

In order to obtain these results, we have made a number of necessary assumptions, the main of which was about the possibility to expand the function, which depends on a “group parameter”, (the functional given on a gauge group), in a series over matrix elements of a special irreducible representation of the gauge group used by Rossi and Testa. That is, we have supposed that there exists an analog of the Peter–Weyl theorem for this representation.

We note that our assumptions appear to be justified, since they should lead to the Schwinger quantum Hamiltonian in the physical subspace of the pure Yang–Mills fields. But the final conclusion may only be done as a result of the examination of these questions.

Path integral transformations in our paper have been performed in integrals defined over the formal measures. In this connection, the basic question, which remains to be answered in further investigations, is an account of the regularization in our transformations (in case of using the regularized metric on a Hilbert manifold) and its possible influence on the final structure of the reduced Hamiltonian.

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8 Appendix

Projection operators and their properties

The projection operator onto the gauge surface Σ can be presented in a following symbolical form:

$$(P_{\perp})_B^A = \delta_B^A - \chi_B^{\mu} (\chi \chi^{\top})^{-1\mu}_{\nu} (\chi^{\top})_{\nu}^A.$$

Let us consider an explicit form of this operator.

The matrix χ_B^{μ} is given by the following formula:

$$\chi_{(\alpha,i,y)}^{(\nu,x)} = \delta_{\alpha}^{\nu} \left[\partial_i(\mathbf{x}) \delta^3(\mathbf{x} - \mathbf{y}) \right].$$

The transposed matrix $(\chi^{\top})_{\mu}^A$, which is defined by equality

$$(\chi^{\top})_{\mu}^A = G^{AB} \gamma_{\mu\nu} \chi_B^{\nu}, \quad \gamma_{\mu\nu} = K_{\mu}^A G_{AB} K_{\nu}^B,$$

has the following form:

$$\begin{aligned} (\chi^\top)^{(\alpha,m,x)}_{(\mu,z)} &= G^{(\alpha,m,x)(\beta,j,y)} \gamma_{(\mu,z)(\alpha,u)} \chi^{(\alpha,u)}_{(\beta,j,y)} \\ &= \left[(\tilde{\mathcal{D}} \cdot \mathcal{D})_\mu^\alpha(\mathbf{z}) \partial^m(\mathbf{z}) \delta^3(\mathbf{z} - \mathbf{x}) \right], \end{aligned}$$

where by $\tilde{\mathcal{D}}$ we denote the operator

$$(-\delta_\mu^\varphi \partial_k(\mathbf{z}) + c_{\sigma\mu}^\varphi A_k^\sigma(\mathbf{z})).$$

The “product” of the matrices $(\chi \cdot \chi^\top)^{(\nu,x)}_{(\mu,z)}$ can be written as follows:

$$\chi^{(\nu,x)}_{(\alpha,m,u)} (\chi^\top)^{(\alpha,m,u)}_{(\mu,z)} = \left[(\tilde{\mathcal{D}}_\mu \cdot \mathcal{D}^\nu)(\mathbf{z}) (-\partial^m(\mathbf{z}) \partial_m(\mathbf{z}) \delta^3(\mathbf{z} - \mathbf{x})) \right].$$

An inverse expression to this matrix product is

$$(\chi \cdot \chi^\top)^{-1(\mu,z)}_{(\alpha,y)} = \int d^3u \left[(\tilde{\mathcal{D}} \cdot \mathcal{D})^{-1} \right]_\alpha^\mu(\mathbf{z} - \mathbf{u}) K(\mathbf{u} - \mathbf{y}),$$

where $K(\mathbf{x} - \mathbf{y})$ satisfies the following equation:

$$(-1) \partial_m(\mathbf{x}) \partial^m(\mathbf{x}) K(\mathbf{x} - \mathbf{y}) = \delta^3(\mathbf{x} - \mathbf{y}).$$

That is, we have

$$(\chi \cdot \chi^\top)^{(\nu,x)}_{(\mu,z)} (\chi \cdot \chi^\top)^{-1(\mu,z)}_{(\alpha,y)} = \delta_\alpha^\nu \delta^3(\mathbf{x} - \mathbf{y}),$$

or

$$(\tilde{\mathcal{D}}^\nu \cdot \mathcal{D}_\mu)(\mathbf{z}) \left[(\tilde{\mathcal{D}} \cdot \mathcal{D})^{-1} \right]_\alpha^\mu(\mathbf{z} - \mathbf{u}) = \delta_\alpha^\nu \delta^3(\mathbf{z} - \mathbf{u}).$$

Thus we get that

$$\begin{aligned} \chi^{(\alpha,x)}_{(\beta,m,z)} \left[(\chi \cdot \chi^\top)^{-1} \right]_{(\alpha,x)}^{(\epsilon,u)} (\chi^\top)^{(\mu,n,y)}_{(\epsilon,u)} = \\ \delta_\beta^\mu \partial_m(\mathbf{z}) \int d^3\tilde{y} K(\tilde{\mathbf{y}} - \mathbf{z}) [\partial^n(\tilde{\mathbf{y}}) \delta^3(\tilde{\mathbf{y}} - \mathbf{y})]. \end{aligned}$$

Taking this into account, we obtain

$$\begin{aligned} (P_\perp)^{(\alpha,k,x)}_{(\beta,m,y)} = \\ \delta_\beta^\alpha \left(\delta_m^k \delta^3(\mathbf{y} - \mathbf{x}) + \partial_m(\mathbf{y}) \int d^3u K(\mathbf{u} - \mathbf{y}) (\partial^k(\mathbf{u}) \delta^3(\mathbf{u} - \mathbf{x})) \right) \end{aligned}$$

Thus, the projection operator can be written symbolically as

$$(P_\perp)^{(\alpha,k,x)}_{(\beta,m,y)} = \delta_\beta^\alpha \left[\delta_m^k + \partial_m \frac{1}{(-\partial^2)} \partial^k \right] \delta^3(\mathbf{y} - \mathbf{x}).$$

In a finite dimensional case, the projector N onto the subspace which is orthogonal to the Killing vector was defined by the following formula:

$$N_B^A = \delta_B^A - K_\alpha^A(Q) (\Phi^{-1})_\mu^\alpha \chi_B^\mu(Q).$$

In our case, it can be written as

$$N_{(\beta,j,y)}^{(\alpha,i,x)} = \delta_{(\beta,j,y)}^{(\alpha,i,x)} - K_{(\mu,z)}^{(\alpha,i,x)} (\Phi^{-1})_{(\nu,u)}^{(\mu,z)} \chi_{(\beta,j,y)}^{(\nu,u)}.$$

An explicit form of this projection operator is

$$N_{(\beta,j,y)}^{(\alpha,i,x)} = \delta_\beta^\alpha \delta_j^i \delta^3(\mathbf{x}-\mathbf{y}) - \mathcal{D}_\epsilon^{\alpha i}(\mathbf{x}) \int d^3u (\Phi^{-1})_{(\beta,u)}^{(\epsilon,x)}(\mathbf{x}-\mathbf{u}) [\partial_j(\mathbf{u}) \delta^3(\mathbf{u}-\mathbf{y})]$$

It can also be written in a symbolical form:

$$N_{(\beta,j,y)}^{(\alpha,i,x)} = \left(\delta_\beta^\alpha \delta_j^i - \mathcal{D}^{\alpha i} \left(\frac{1}{\mathcal{D}} \frac{\partial}{\partial} \right)_\beta \partial_j \right) \delta^3(\mathbf{x}-\mathbf{y}).$$

The main properties of these projection operators are as follows:

$$\begin{aligned} N_{(\beta,j,y)}^{(\alpha,i,x)} N_{(\mu,k,z)}^{(\beta,j,y)} &= N_{(\mu,k,z)}^{(\alpha,i,x)} \\ N_{(\beta,j,y)}^{(\alpha,i,x)} K_{(\mu,z)}^{(\beta,j,y)} &= 0 \\ N_{(\beta,j,y)}^{(\alpha,i,x)} (P_\perp)_{(\alpha,i,x)}^{(\mu,k,z)} &= N_{(\beta,j,y)}^{(\mu,k,z)} \\ (P_\perp)_{(\beta,m,y)}^{(\alpha,k,x)} N_{(\alpha,k,x)}^{(\epsilon,n,z)} &= (P_\perp)_{(\beta,m,y)}^{(\epsilon,n,z)} \\ (P_\perp)_{(\beta,m,y)}^{(\alpha,k,x)} (P_\perp)_{(\epsilon,n,z)}^{(\beta,m,y)} &= (P_\perp)_{(\epsilon,n,z)}^{(\alpha,k,x)} \\ (P_\perp)_{(\beta,m,y)}^{(\alpha,k,x)} (\chi^\top)_{(\mu,z)}^{(\beta,m,y)} &= 0 \\ (P_\perp)_{(\beta,m,y)}^{(\alpha,k,x)} \chi_{(\alpha,k,x)}^{(\nu,z)} &= 0 \\ N_{(\beta,j,y)}^{(\alpha,i,x)} \chi_{(\alpha,i,x)}^{(\nu,z)} &= 0. \end{aligned}$$

References

- [1] L. D. Faddeev, V. N. Popov, *Phys. Lett. B* **25** (1967) 29;
L. D. Faddeev, *Theor. Math. Phys.* **1** (1970) 1.
- [2] G.C. Rossi, M. Testa, *Nucl.Phys.* **B163** (1980) 109;
B176 (1980) 477.

- [3] C. Teitelboim, *J. Math. Phys.* **25** n.4, (1984) 1093–1101.
- [4] H. Reinhardt, *Mod. Phys. Lett.* **A11** (1996) 2451; *Phys. Rev.* **D55** (1997) 2331.
- [5] S. N. Storchak, *Bogolubov transformation in path integrals on manifold with a group action*. IHEP Preprint 98–1, Protvino, 1998;
S. N. Storchak, *J. Phys. A: Math. Gen.* **34** (2001) 9329;
S. N. Storchak, *Physics of Atomic Nuclei* **64** n.12 (2001) 2199.
- [6] S. N. Storchak, *J. Phys. A: Math. Gen.* **37** (2004) 7019–7038.
- [7] G. Kunstatter, *Class. Quant. Grav* **9** (1992) 1466–1486.
- [8] N. P. Landsman, N. Linden, *Nucl. Phys.* **B365** (1991) 121;
S. Tanimura, I. Tsutsui, *Mod. Phys. Lett* **A34** (1995) 2607;
D. McMullan, I. Tsutsui, *Ann. Phys.* **237** (1995) 269.
- [9] I. M. Singer, *Physica Scripta* **24** n.5 (1981) 817;
I. M. Singer, *Commun. Math. Phys.* **60** (1978) 7.
- [10] M. S. Narasimhan, T. R. Ramadas, *Commun. Math. Phys.* **67** (1979) 121.
- [11] D. Groisser, T. H. Parker, *J. Diff. Geom.* **29** (1989) 499.
- [12] Yu. P. Soloviev, Geometrical structures on a manifold of interacting gauge fields, *Global analysis and mathematical physics* (Voronezh, 1987) 110–121 (in Russian).
- [13] M. Asorey, P. K. Mitter, *Commun. Math. Phys.* **80** (1981) 43.
- [14] P. K. Mitter, C. M. Viallet, *Commun. Math. Phys.* **79** (1981) 457.
- [15] Yu. L. Daletskii, *Usp. Mat. Nauk* **22** n.4 (1967) 1 (in Russian).
- [16] Ya. I. Belopolskaya, Yu. L. Daletskii, *Russ. Math. Surveys* **37** 109 (1982);
Usp. Mat. Nauk **37** n.3 (1982) 95 (in Russian);
Yu. L. Daletskii, *Usp. Mat. Nauk* **38** n.3 (1983) 87 (in Russian);
Ya. I. Belopolskaya and Yu. L. Daletskii, *Stochastic equations and differential geometry* (Kluwer, Dordrecht, 1990), Mathematics and Its Applications, Soviet Series, 30.
- [17] B. Gaveau, Ph. Trauber, *C. R. Acad. Sci. A, Paris* **t29** (1980) 673;
J. Funct. Anal. **38** (1980) n.3 324.

- [18] B. Gaveau, Ph. Trauber, *C. R. Acad. Sci. A, Paris* **291** (1980) 575.
- [19] Yu. M. Berezansky, Yu. G. Kondratiev, *Spectral methods in infinite dimensional analysis*, (Kluwer Academic Publishing, Holland, 1995).
- [20] S. Albeverio, R. Høegh-Krohn, *Physics Reports* **77** (1981) 193.
- [21] U. Semmler, *J. Math. Phys.*, **30** (1989) 1957.
- [22] S. Paycha, *Commun. Math. Phys.* **166** (1995) 433.
- [23] M. Arnaudon, S. Paycha, *Commun. Math. Phys.* **187** (1997) 243.
- [24] O. A. Khrustalev, A. V. Razumov, A. Yu. Taranov, *Nucl. Phys.* **B172** (1980) 44;
 A. V. Razumov, A. Yu. Taranov, *Teor. i Mat. Fiz.* **52** (1982) 34 (in Russian);
 A. V. Razumov, Bogolubov Transformation and Quantum Theory of Constrained Systems, *Doctor of Science Thesis* (Protvino, 1991) (in Russian).
- [25] N. H. Christ, T. D. Lee, *Phys. Rev. D* **22** (1980) 939.
- [26] M. Creutz, I. Muzinich, T. N. Tudron, *Phys. Rev. D* **19** (1979) 531;
 M. Creutz, *Phys. Rev. D* **15** (1977) 1128;
 T. N. Tudron, *Phys. Rev. D* **21** (1980) 2348.
- [27] N. K. Falck, A. C. Hishfeld, *Ann. of Phys.* **144** (1982) 34.
- [28] R. S. Lipster and A. N. Shiryaev, *Statistics of Random Processes* (Springer-Verlag: Berlin, Heidelberg, New York, 1977), Vols. I, II.
- [29] V. S. Pugachev and I. N. Sinitsyn, *Stochastic differential systems*, (Nauka, Moscow, 1990, 2nd ed.).
- [30] R. Abraham, J. E. Marsden, *Foundation of Mechanics, 2nd Ed.* (Addison-Wesley Redwood City, 1985).
- [31] Yu. L. Daletskii, N. I. Teterina, *Usp. Mat. Nauk* **27** n.2 (1972) 167–168; (in Russian);
 Yu. L. Daletskii, *Usp. Mat. Nauk* **38** n.2 (1975) 209–210 (in Russian)